

MATH4022A Test 1, Sept. 19, 2001

Name :

1. (a) (5 points) How many spanning trees does this graph have?

(b) (5 points) Give a sequence of Δ -reversals (reversals of directed 3-cycles) which transforms T into T' (or T' into T).

2. (5 + 5 points) Let T be a tree on n vertices, for $n \geq 2$. Let $\Delta(T)$ denote the maximum degree in T . (a) Show that T has at least $\Delta(T)$ leaves. (b) When does T have exactly 2 leaves? When does T have exactly $\Delta(T)$ leaves?

3. (4 + 6 points) Consider the Prüfer code for the labeled spanning trees of K_n , for $n \geq 3$. Let the vertices be labeled $1, 2, \dots, n$. Let S be a spanning tree of K_n .

(a) While constructing the list $(a_1, a_2, \dots, a_{n-2})$ corresponding to the tree S , does vertex n ever get deleted from the tree? (explain your answer.)

(b) Prove or disprove the following: A spanning tree S contains the edge $\{n-1, n\}$ if and only if the last item a_{n-2} is $(n-1)$ or n in the list $(a_1, a_2, \dots, a_{n-2})$ corresponding to the tree S .