MATH 4022A Test 2, Oct. 19, 2001

Name :

You may turn in solutions to the last problem (for extra credit) on Monday at 10:00 am.

1. (5+5 points)

   (a) Let T be a minimum weight spanning tree in a weighted connected graph G. Prove or disprove that T omits some heaviest edge from every cycle in G.

   (b) Find a minimum (weight) spanning tree in the following graph:

2. (4 + 6 points)

(a) A line of a matrix is a row or a column of the matrix. Show that the minimum number of lines containing all the 1's of a (0,1)-matrix is equal to the maximum number of 1's, no two of which are in the same line.

(b) Let G be a 3-regular graph with at most 2 cutedges. Prove that G has a perfect matching. (Hint: slightly modify the proof shown in class for the case of no cutedges.)

3. (10 points)

   Use the König-Egerváry theorem to prove that every bipartite graph G has a matching of size at least $e(G)/\Delta(G)$. Use this to conclude that every subgraph of $K_{n,n}$ with more than $(k-1)n$ edges has a matching of size at least k.

4. (Extra Credit)

(a) Prove that every graph has a matching of size at least $n/(1+\Delta(G))$. (Hint: Use induction on the number of edges of G.)

(b) Let G be an X, Y-bigraph having a matching that saturates X. Let S and T be subsets of X such that $|N(S)| = |S|$ and $|N(T)| = |T|$. Prove that $|N(S \cap T)| = |S \cap T|$.