Course: MATH 4032 Combinatorial Analysis (Spring'07) – Homework 3

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Office Hours: **Mon, Wed 11am – noon, Thurs. 2:00 – 3:00pm**

Due: No need to turn in

1. Solve $h_n = 5h_{n-1} - 6h_{n-2}$, for $n \geq 2$, using the generating function approach; you may use $h_0 = 1,\ h_1 = -2$, for initial conditions.

2. Determine the exponential generating function for the sequence of factorials: $0!, 1!, 2!, \ldots, n!, \ldots$

3. Solve for h_n using the generating functions (or any other method) $h_n = 3h_{n-2} - 2h_{n-3}$, $(n \geq 3)$; $h_0 = 1, h_1 = h_2 = 0$.

4. Determine the generating function for the sequence of cubes: $0, 1, 8, \ldots, n^3, \ldots$

5. Prove that $R(4, 3) \leq 10$.

6. Let $x_1, x_2, \ldots, x_n$ be real numbers, $x_i \geq 1$ for each i, and let S be the set of all numbers, which can be obtained as a linear combination $\alpha_1 x_1 + \cdots + \alpha_n x_n$ with $\alpha_i \in \{+1, -1\}$. Let $I = [a, b)$ be any interval (in the real line) of length $b - a = 2$. Show that $|I \cap S| \leq \binom{n}{n/2}$.

(*Hint*: Associate with each sum $s = \alpha_1 x_1 + \cdots + \alpha_n x_n$ the corresponding set $A_s = \{i : \alpha_i = +1\}$. Observe that the family of sets A_s, for which $s \in I$, forms an antichain.)

Useful Binomials:

$$(i) \quad \binom{-n}{k} = (-1)^k \binom{n+k-1}{k} .$$

$$(ii) \quad \frac{1}{(1-rx)^n} = \sum_{k=0}^{\infty} (-1)^k \binom{-n}{k} r^k x^k = \sum_{k=0}^{\infty} \binom{n+k-1}{k} r^k x^k , \qquad (|x| < \frac{1}{|r|}) .$$