MATH 4032 Combinatorial Analysis (SPR'07) – Homework 7

Instructor : Prasad Tetali, office: Skiles 234, email: tetali@math.gatech.edu
Office Hours: **Mon. Wed. 11–12, Thurs. 1:30–2:30pm**

Due: next Monday

1. Prove that in every 3-coloring of the edges of K_{17}, there is a monochromatic triangle.
Hint: Argue as in the proof of $R(3,3) \leq 6$.

2. Prove that for all integers $r \geq 1$, there is a minimal number $N(r)$ with the following property. If $n \geq N(r)$ and the integers in $\{1, 2, \ldots, n\}$ are colored with r colors, then there are three elements x, y, z (not necessarily distinct) with the same color $x + y = z$.

Hint: Take $\{1, 2, \ldots, n\}$ as the vertex set, and color each edge $\{i, j\}$ using the color of $|i - j|$. Appropriate Ramsey theorem tells us that there is a triangle whose edges all get the same color. So ...?

3. Let $W(r, k)$ be the least number n such that any coloring of $\{1, 2, \ldots, n\}$ using r colors gives a monochromatic k-term arithmetic progression. Provide a lower bound for $W(r, k)$, for general $r \geq 2$, using the Lovász local lemma. Compare your bounds with the ones obtained using the basic probabilistic method.

4. Recall that a *clause* is an Or of literals, where each literal is a boolean variable x_i or its negation $\bar{x}_i$. Let $C_1, C_2, \ldots, C_m$ be a collection of clauses, each consisting of k literals, and such that every variable appears (negated or not) in at most r clauses. Assume that $r < 2^{k-2}/k$. Prove that then there is an assignment which satisfies all the clauses $C_1, C_2, \ldots, C_m$.

5. Prove that $R(t, t) \leq \binom{2t-2}{t-1}$.
Hint: Use the upper bound $R(s, t) \leq R(s - 1, t) + R(s, t - 1)$.