Name: _____

Math 4150 – Introduction to Number Theory Spring 2010

Test # 1 (Make-up)

You have 50 minutes to take this test. There are 52 total points possible. Simple calculators are fine, but no programmable calculators are allowed. *Please write your answers neatly and show all of your work.* Feel free to write on both sides of each sheet.

1. (6+6+5=17 points) Answer each of the following with one or two sentences of explanation.

   a. If $ab \equiv ac \pmod{n}$ then $b \equiv c \pmod{n}$. True or False?
   If $a \equiv b \pmod{n}$ and $c \equiv d \pmod{n}$ then $ad \equiv bc \pmod{n}$. True or False?

   b. Let p be prime and a, m and n be positive integers. Suppose $m \equiv n \pmod{p-1}$. Then explain why $a^m \equiv a^n \pmod{p}$.

   c. For large positive integer n, roughly how many integers between 1 and n are prime numbers?

2. (10 points) Show that there are infinitely many primes of the form $6k+5$.

3. (5+10=15 points)

   a. State the Chinese Remainder Theorem.

   b. What is the smallest number of lobsters in a tank if 1 lobster is left over when they are removed 2, 3, 5, or 7 at a time, but no lobsters are left over when they are removed 11 at a time?

4. (4+6=10 points) *Note that the two parts of this problem have nothing to do with each other.*

   a. When does $ax \equiv b \pmod{n}$ have an integer solution in x, given that a, b and n are integers? Explain how you would find a solution.

   b. What can you conclude if a, b, and c are positive integers such that $(a, b) = (b, c) = 1$ and $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$ is an integer?

Extra Space