## Course: Math 6221 - Homework 1 (Fall 2005)

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Due: Thursday, Sept. 1st

The following exercises are all from the Grimmett-Stirzaker book (3rd ed., 2001). Please stop by my office, if you don't have a copy of the book. I can give a copy of the problems.

Problem 1. Exercise 1.4.5
(This is just to test basic knowledge of conditional probability, and since this is a classic problem.)

Problem 2. Exercise 1.8.16

Problem 3. Exercise 1.8.17

Problem 4. Exercise 1.8.18

Problem 5. (a) Let $\Omega$ be a set and let $A_{1}$ and $A_{2}$ be subsets of $\Omega$. Show that the smallest $\sigma$-algebra containing $A_{1}$ and $A_{2}$ consists of at most 16 sets.
(b) Let $A_{1}, A_{2}, \ldots, A_{k}$ be subsets of $\Omega$. Let $\mathcal{F}_{\|}$be the smallest $\sigma$-algebra containing the $A_{i}$ 's. Show that $\mathcal{F}_{\|}$has at most $2^{2^{k}}$ members.
(c) Show that the upper bound in part (b) can not be improved. (Hint: Let $M$ be the $k$-element set $\left\{p_{1}, \ldots, p_{k}\right\}$, and let $\Omega=2^{M}$ be the set of subsets of $M$. Let $A_{i}$ be all subsets of $M$ that contain the point $p_{i}$.

