Course: Math 6221 Fall'05 – Homework 3

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Office Hours: Mon. 11-12am, Tue, Thur. 2-3pm

Due: Thursday, Oct. 13th

Problem 1. Two people plan to meet between 5pm and 6pm, each agreeing not to wait more than 10 minutes for the other. Find the probability that they will meet if they arrive independently at (uniformly) random times between 5pm and 6pm.

Problem 2. In your pocket is a random number N of coins, where N has the Poisson distribution with parameter λ. You toss each coin once, with heads showing with probability p each time. Show that the total number of heads has the Poisson distribution with parameter λp.

Problem 3. Find the characteristic function corresponding to each of the probability density functions.

(a) $f(x) = \frac{a}{2}e^{-a|x|}$

(b) $f(x) = \frac{a}{\pi(a^2+x^2)}$

Problem 4. Let X be an absolutely continuous random variable having the density $f_X(x) = \frac{1}{2}e^{-|x|}, -\infty < x < \infty$. Find the moment generating function $M_X(x)$, and use it to compute all the moments EX^n, for $n \geq 1$.

Problem 5. (a) Suppose that $X_n \to X$ in 1st mean. Show that $EX_n \to EX$. Is the converse necessarily true?

(b) Show that $X_n \to X$ almost surely whenever

$$\sum_n E(|X_n - X|^r) < \infty \text{ for some } r > 0.$$

Problem 6. Let $X_1, X_2, \ldots$ be bounded, independent, identically distributed random variables with mean zero. Let $S_n = \sum_{i=1}^n X_i$. Show that if $\alpha > 0$ then almost surely,

$$\frac{S_n}{n^{(1/2)+\alpha}} \to 0, \quad \text{as } n \to \infty.$$

(*Hint*: Show that there exists a constant C_k such that

$$E(S_n^{2k}) \leq C_k n^k,$$

for every integer $k > 0$.)

Problem 7. Prove that as $n \to \infty$

$$e^{-n} \sum_{k=0}^n \frac{n^k}{k!} \to 1/2.$$

(*Hint*: Consider sums of independent Poissons with mean one.)