Course: Math 6221 – Advanced Classical Probability Theory

Instructor : Prasad Tetali, office: Skiles 234, email: tetali@math.gatech.edu

Office Hours: To be anounced (Fall 2005)

Course Objective.

• This is a special course in probability designed and required for the Ph.D. students in the ACO program. The objective is to provide a rigorous intro to classical probability theory (including the laws of large numbers and the central limit theorem), and to advance to the modern theory (including coupling, martingales, and functional techniques).

Suggested Books : Class Notes plus

• Probability and Random Processes, 3rd ed. by Grimmett and Stirzaker (Oxford Science Publications, 2001).

- An introduction to probability and stochastic processes, by Marc Berger (Springer-Verlag).
- Measure theory and Probability, by Adams and Guillemin, published by (Birkhäuser, 1996).

Outline of Syllabus.

- Random variables and distributions (Discrete and Continuous)
- Coupling and Poisson approximation

• Measure and Integration (Lebesgue Integration, Monotone convergence theorem, Dominated convergence theorem, Fubini's theorem)

• Expectation and modes of convergence (Conditional expectation, Markov, Chebychev, Hölder, Minkowski and other inequalities, various notions of convergence of random variables)

• Characteristic functions, moment generating functions, and the central limit theorem

• Laws of large numbers (Borel-Cantelli lemmas, Kolmogorov three series theorem, Kolmogorov's strong law)

• Conditional Expectation and Martingales (Basics, Optional stopping, Azuma-Hoeffding and Talagrand inequalities)

• Uniform Integrability and UI Martingales

• Finite Markov chains (Stationary measure, convergence to stationarity, spectral gap and spectral profile)

Optional:

- Functional inequalities (Poincaré, Log-Sobolev, Transportation) and applications
- \bullet Random graphs \dots

Test 1: Thursday, September 22nd Test 2: Tuesday, November 1st

No make-up tests will be allowed. Homeworks will be assigned and collected.

General grading policy: Homeworks 30%, Tests 40%, Final exam 30%