Math 7018 Spring 2006
Assignment 1. Due Wednesday, January 25th, 2006

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1. **Property B**: Show that $m(3) > 6$, using the strategy outlined in the class: first show that we may assume that the number of points is **at most** 9 and then assume that we have exactly 9 points, by adding extra ones as dummy points, not included in any set; finally use a random equicoloring: choose a random coloring with 5 Reds and 4 Blues uniformly over all such colorings.

2. The Bipartite Ramsey Number $BR(k)$ is the least n so that if A, B are disjoint with $|A| = |B| = n$ and $A \times B$ is two-colored (meaning the edges between A and B are two-colored), there exist $A_1 \subseteq A, B_1 \subseteq B$ with $|A_1| = |B_1| = k$ and $A_1 \times B_1$ monochromatic. Find and prove a theorem which gives a lower bound for $BR(k)$ and explore the asymptotics.

3. **Exercise 2 from the book**: Suppose $n \geq 4$ and let H be an n-uniform hypergraph with at most $\frac{4^{n-1}}{3^n}$ edges. Prove that there is a coloring of the vertices of H by four colors so that in every edge all four colors are represented.

4. **Exercise 8 from the book**: (Prefix-free codes; Kraft inequality). Let F be a finite collection of binary strings of finite lengths and assume no member of F is a prefix of another one. Let N_i denote the number of strings of length i in F. Prove that
$$\sum_i \frac{N_i}{2^i} \leq 1.$$

5. Find $m = m(n)$ as large as you can so that the following holds: Let $A_1, \ldots, A_m \subseteq \{1, \ldots, 4n\}$ with all $|A_i| = n$. Then there exists a two coloring of $\{1, \ldots, 4n\}$ such that none of the A_i are monochromatic. Use a random equicoloring of $\{1, \ldots, 4n\}$. Express your answer as an asymptotic function of n.