

Course: Math 7018 – Probabilistic methods in combinatorics

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Office Hours: TBA

Suggested Textbook: **The Probabilistic Method**, by N. Alon and J. Spencer (with an appendix of problems by Paul Erdos), 2nd ed. (2000), John Wiley & Sons.

Additional handouts will be given based on material from other sources, some of which are listed below.

References:

- “Random Graphs,” by Janson, Luczak, and Rucinski, John Wiley & Sons (2000).
- “Probabilistic Methods for Algorithmic Discrete Mathematics,” Ed. by M. Habib et al. (1998), Springer-Verlag.
- “The Handbook of Combinatorics” (e.g. the chapter on *Combinatorial discrepancy theory* by J.Beck and V. Sos)

Course Objective. To develop an appreciation for the strength and beauty of the probabilistic techniques in combinatorics.

Rough Outline.

1. *The basic (first moment) method:* Examples from graph theory, combinatorics, and number theory, of the use of the probabilistic method, the use of linearity of expectation
2. *The second moment method:* Number-theoretic and Random graph applications.
3. *The Lovasz local lemma:* Basic lemma, its variations and applications
4. *Alterations:* Ramsey numbers, Property B, Packing and Recoloring
5. *Combinatorial discrepancy theory:* Balancing lights, Spencer’s six standard deviations result, Beck-Fiala theorem and the Komlos conjecture
6. *Correlation inequalities:* The four functions theorem, FKG and XYZ inequalities
7. *Random graphs:* Chromatic number, Clique number, Sharp threshold phenomenon
8. *The Poisson paradigm:* Janson’s inequalities
9. *Martingale Inequalities:* Azuma-Hoeffding inequality, Talagrand’s inequality
10. *Entropy techniques:* Shearer’s lemma and combinatorial enumeration applications
11. *Pseudorandomness and Derandomization.* Eigenvalues and Expanders.

Grading policy: Homeworks will be assigned and graded. There will be two Tests. Students (in pairs) may have to read and present a research article, relevant to the course.