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Math 3012 – Applied Combinatorics
Lecture 1

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Syllabus, Grading System and Test Dates

Please consult the detailed syllabus, explanation of the grading system and schedule of tests posted on the course web site. These materials are accessible from my Georgia Tech home page. Grades and email archive will be on T-Square. Please note that the final exam date is **firm** and is set for Tuesday, December 8, 2015 at 8:00am here in MRDC.

Attendance Policy

I will do everything possible to make regular attendance stimulating and rewarding, perhaps even enjoyable. Some quizzes will include material that can only be answered if you have been present and paying attention. Nevertheless, if you get 100% on all tests and never come to class, you will still get an A+. I think you will still get less than you paid for, but …

However, please remember that I have a **very** strict policy regarding excused absences from tests and the final exam. Oversleeping, the student health center and such will not be accepted as valid reasons for missing a test.

Homework Assignments

Homework Assignments will be made on a regular basis and solutions collected at least twice, perhaps three times during the semester. But please do not depend on this channel as an on-line feedback system. Human resource issues dictate that almost certainly, feedback will occur about the same time as tests, so use office hours, in class questions, etc., to make sure you are on target and are current with understanding principles and methods.

Two Groups of Students

This course will be taught in a regular classroom setting on the Georgia Tech campus during the Fall semester of 2015. However, the lectures are being video taped, and after some judicious editing, they will be posted on-line for students in this section (and other sections being taught concurrently) to view.

However, the edited versions will also be used in subsequent terms for students enrolled in Math 3012 via distance learning. For this reason, there will be some obvious moments where assignments, questions, comments and such will be addressed with two audiences in mind.

Getting Started

Are these two sequences the same?

S = 1010110110101110110000011000101010100

T= 1010110110101110100000011000101010100

Slightly Harder Problem

Question I have two DVD's which are supposed to be identical installation disks for the new Windows 10. How can I be sure that they are indeed identical?

Fair Division

Example Given the numbers:

12 17 22 31 48

We observe that 12 + 22 + 31 = 17 + 48.

Question Can you find a fair division of the numbers:

46 63 77 85 91 102 113 142 168 184 192 210
240 253 267 295 304 322 339 360 381 399
401 439 444 467 482 492 520 531 552

Slightly Harder Problem

Question I have a 1.44 mb floppy disk full of integers. Can I be certain that the total computing power on the planet is enough to settle the fair division problem for this set of numbers?

Remark Some of you are too young to remember floppy disks, but if you will come to my office, I will show you one!

Factoring

Question Is the following number a prime?

121722314823006489210074712319099317259

Factoring

Question Is the following number a prime?

121722314823006489210074712319099317259

Question If I told you the answer is no, would you believe me?

Factoring

Question Is the following number a prime?

12172231482300648921007471231909931 7259

Question If I told you the answer is no, would you believe me?

Question If I told you that this number is divisible by 11 and 293, can you find the other factors?

Output from Maple

Solution Here is factorization provided by Maple:

121722314823006489210074712319099317259 =

(11)(293)(37766774689111538693786755296028333)

Note You could find the third factor by hand calculations in less than 10 minutes. How long would it take you to confirm that it is a prime?

Slightly Harder Problem

Question What fate would befall someone who is reliably able to factor integers with 200 digits? What about 200,000,000,000,000,000,000 digits?

Question Who cares?

Question Why am I paying money to think about such silly questions?

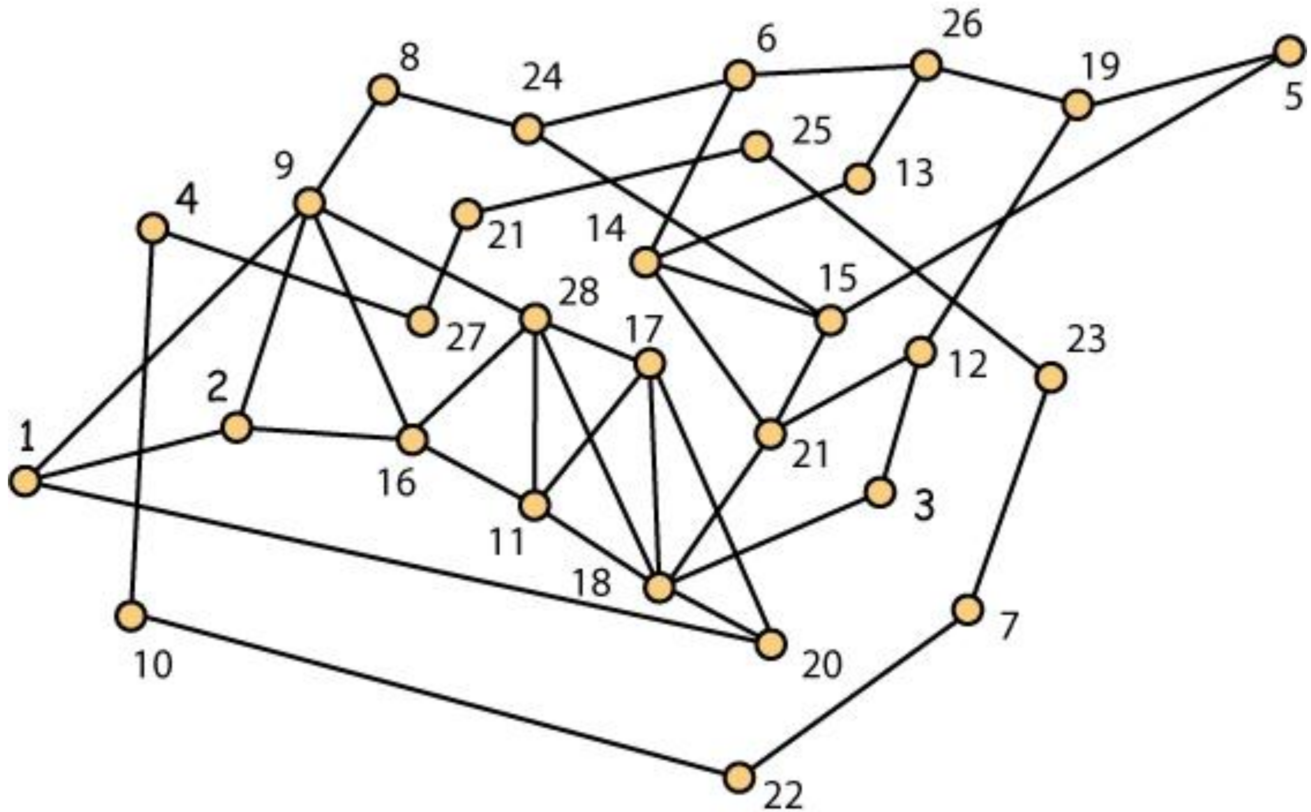
Adding Fractions

Remark In elementary school, students are taught to add fractions by finding least common multiples.

Question How would you add the following fractions?

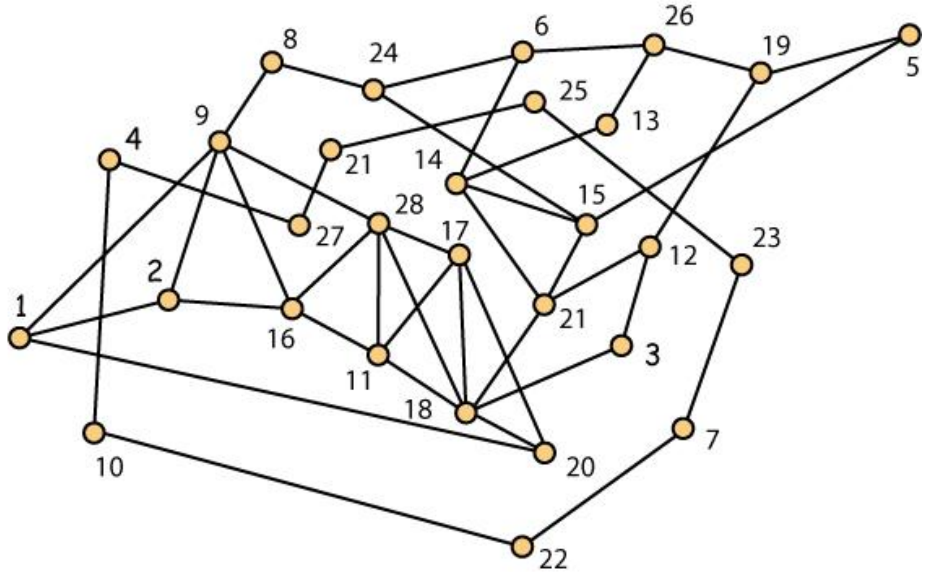
$$\frac{1217223148}{42399006718} + \frac{23006489210074712}{890204476285434426}$$

Graphs



Natural Questions

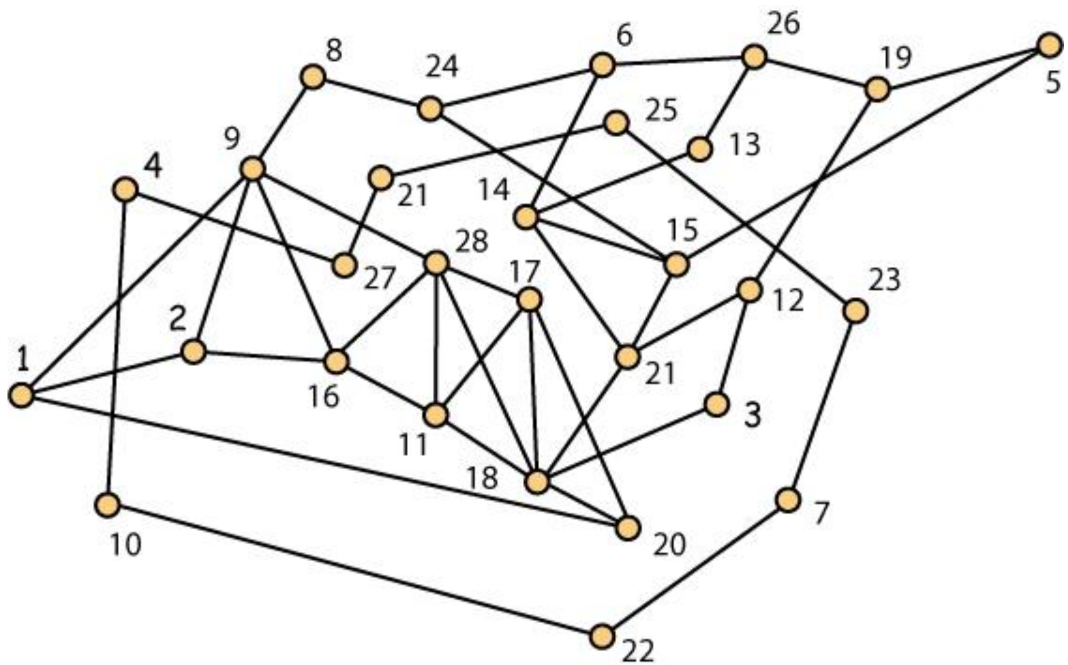
Question How many vertices does this graph have? How many edges?



Remark In fact, there are 29 vertices, since the label 21 is mistakenly used twice!! So be careful!

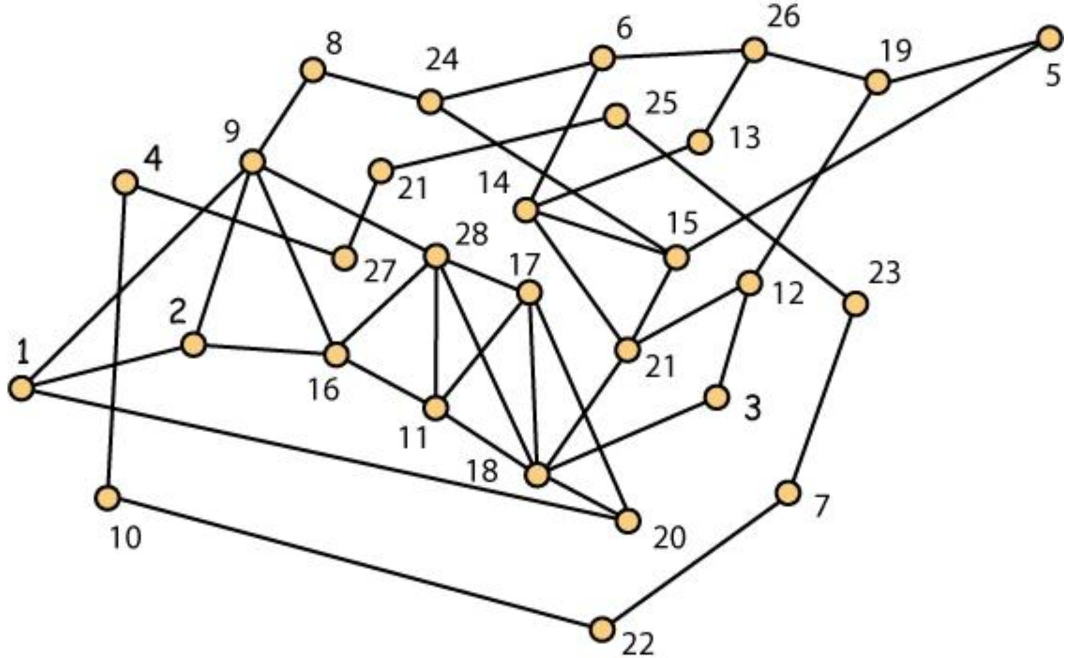
Natural Questions

Question Can you draw this graph without crossings?



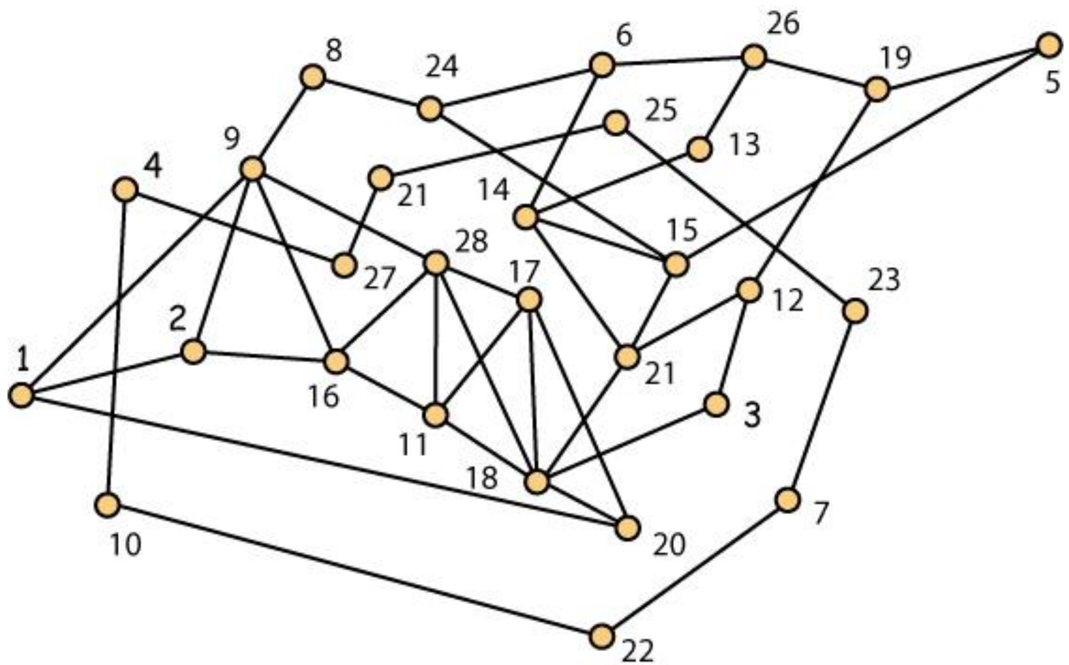
Natural Questions

Question What is the length of the shortest path from 1 to 12?



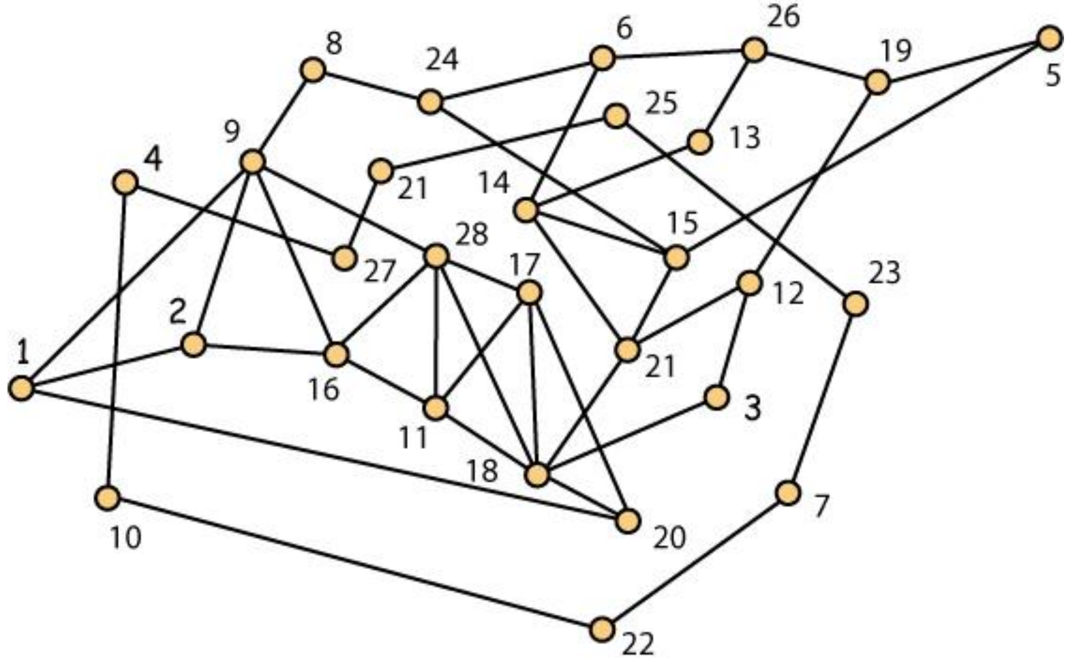
Natural Questions

Question Is this graph connected?



Natural Questions

Question Starting from 1, and walking only along edges, what is the maximum number of vertices you can visit without visiting any vertex more that once?



Slightly Harder Problem

1. What is a good way to convey essential information for a graph with 1,579,200 vertices?
2. Given such a graph, do you have any chance of determining whether it can be drawn without edge crossings?
3. Can you determine whether it is connected?
4. Can you find the maximum distance between two vertices?
5. Can you determine whether there is a way to visit each vertex exactly once, walking only on edges? Assume access to a super computer.

Very Practical Problem

Question Within five years of "getting out" from Georgia Tech, would you rather your annual salary in US dollars be

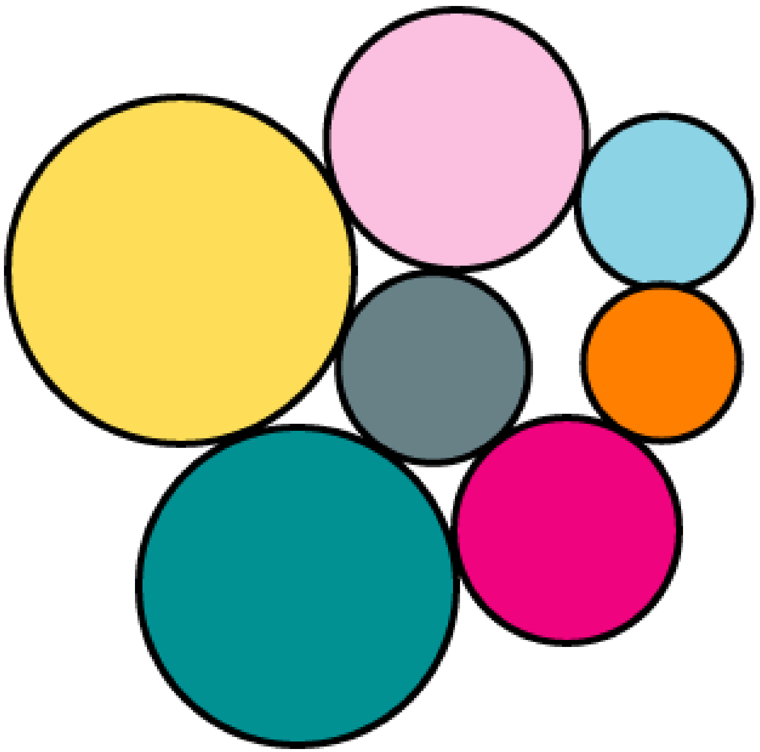
2^{10^4} or $100{,}000 \times 100{,}000$?

Another Practical Problem

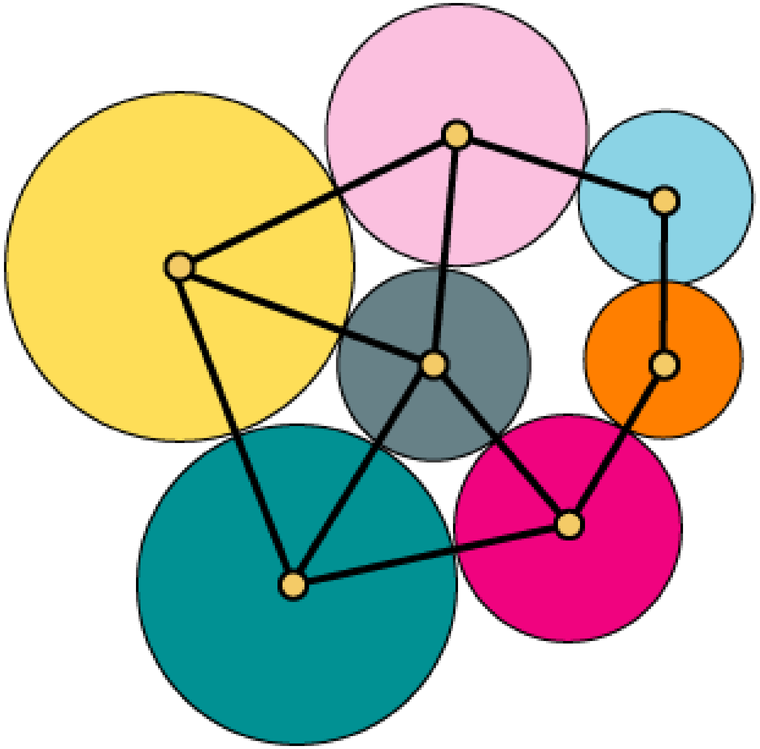
Question Looking to the future, now that you have graduated and achieved considerable fame, success and wealth, your generous gift back to Georgia Tech (earmarked for the the School of Mathematics, of course) will be which of the following amounts in US dollars?

2^{10^2} or 1024×1024

Families of Disks



“Kissing Coins” Graphs



The Kissing Coins Theorem

Theorem (Koebe, 1936) Every graph that can be drawn without crossings has a representation as a set of ``kissing coins".

Trust me, it's true!!

Question: Hard or Easy?

Question Alice gives Bob the data for a graph on 192, 674 vertices and tells him that she knows that the graph can be drawn without crossings. She then challenges Bob to find a ``kissing coins" representation. Is this a fair challenge?

Tantalizing Questions

1. Are there theorems that can be easily stated but for which any proof must be very long?
2. How can one easily distinguish between easy problems and hard problems?
3. Alice knows she is right. How can she convince Bob?
4. Bob believes that Alice is wrong. How can he convince her?
5. What is the precise meaning of the following words: Big, small, difficult, easy, doable, impossible, long, short.