Student Name and ID Number

MATH 3012, Quiz 3, November 25, 2014, WTT

1. a. In terms of partitions of an integer, interpret the meaning of the coefficient of x^{47} in the following generating function:

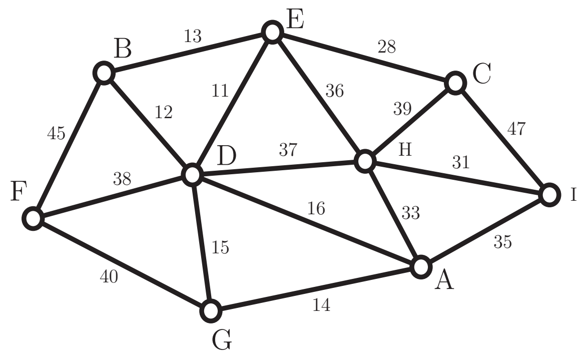
$$f(x) = \frac{1 + x^2 + x^3}{(1 - x^7)(1 - x^{14})}$$

b. Without doing any multiplication of polynomials, find the value of the coefficient of x^{47} in $f(x)$.

2. a. An advancement operator equation has the form $p(A)f = 0$ where $p(A)$ is a polynomial and $f = f(n)$ is a function from integers to the field of complex numbers. If you know that $2 - 3i$ is a root of multiplicity 4 of the polynomial $p(A)$, what terms in the basis for the solution space are associated with this root?

b. Find the solution to the recurrence equation: $r_n = r_{n-1} + 20r_{n-1}$, subject to the initial conditions: $r_0 = 15$ and $r_1 = 12$. You may use either advancement operator equations or generating functions.

3. A graph with weights on edges is shown below. List *in order* the edges which make up a minimum weight spanning tree using, respectively, Kruskal's Algorithm (avoid cycles) and Prim's Algorithm (build tree). For Prim, use vertex A as the root.



Kruskal's Algorithm

Prim's Algorithm

4. Let P be a poset whose ground set is $\{a, b, c, d, e, f, g, h\}$. A bipartite matching algorithm is used to solve the Dilworth problem for this poset. When the algorithm halts, the edges in the maximum matching are listed below on the left.

a. In the space to the right, assemble the chain partition associated with this matching.

$a'g''$
$e'f''$
$f'b''$
$g'h''$
$d'a''$

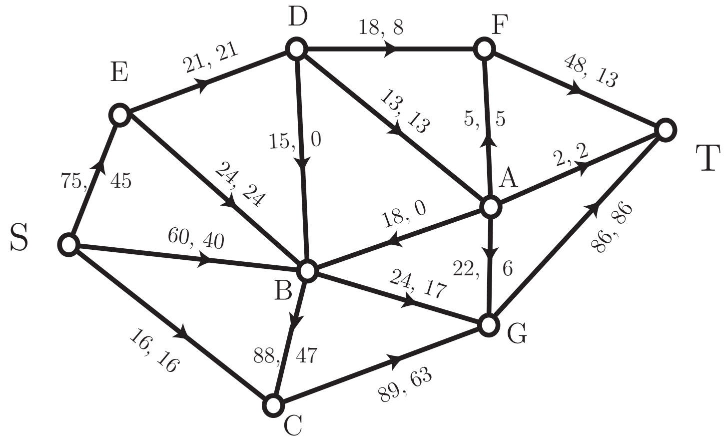
b. What is the width of the poset P.

c. Explain how you would use the labelling information obtained when the network flow (bipartite matching) algorithm halts to find a maximum antichain in P.

5. A data file `digraph_data.txt` has been read for a digraph whose vertex set is [7]. The weights on the directed edges are shown in the matrix below. The entry $w(i, j)$ denotes the length of the edge from i to j. If there is no entry, then the edge is not present in the graph. Apply Dijkstra's algorithm to find the distance from vertex 1 to all other vertices in the graph. Also, for each x, find a shortest path from 1 to x. Please show your work.

W	1	2	3	4	5	6	7
1	0	38	42	17	38	64	29
2		0	4		30	23	10
3			0		41	18	
4	27	20	28	0		45	
5			2		0	21	9
6	82	5	3	2	2	0	
7		8	22	4	18	12	0

6. Consider the following network flow:



a. What is the current value of the flow?

b. What is the capacity of the cut $V = \{S, B, D, E\} \cup \{T, A, C, F, G\}$.

c. Carry out the labeling algorithm, using the pseudo-alphabetic order on the vertices and list below the labels which will be given to the vertices.

d. Use your work in part c to find an augmenting path and make the appropriate changes directly on the diagram.

e. Carry out the labeling algorithm a second time on the updated flow. It should halt without the sink being labeled.

f. Find a cut whose capacity is equal to the value of the updated flow.

7. True–False. Mark in the left margin.

1. Prim's algorithm only works when the root is incident with an edge of minimum weight in the graph.
2. To implement Kruskal's algorithm, it is not necessary to sort the edges by weight. One can simply take the edges in any order and take the first one avoiding a cycle when added to those edges already chosen.
3. Dijkstra's algorithm finds shortest paths having the maximum number of edges.
4. The key idea behind the Ford-Fulkerson algorithm for network flows is to find at each step an augmenting path which maximizes the increase in the amount of the flow.
5. All linear programming problems posed with integral constraints have integral solutions.
6. A bipartite graph $G = (X, Y, E)$ with $|X| = 400$, $|Y| = 600$ and $|E| = 10,000$ always has a matching of size 200.
7. When P is a poset on 100 points and the height of P is 15, there is always a partition of P into 15 chains.
8. Generating functions spanning Kruskal flows with rational paths always determine shortest coefficients even when Dijkstra's rose bush is Primed.