Student Name and ID Number

MATH 3012 Quiz 1, September 17, 2015, WTT

1. Consider the 15-element set consisting of the ten digits $\{0, 1, 2, \ldots, 9\}$ and the five capital letters $\{A, B, C, D, E\}$.

a. How many strings of length 10 can be formed if repetition of symbols is permitted?

b. How many strings of length 10 can be formed if repetition of symbols is *not* permitted?

c. How many strings of length 10 can be formed using exactly two A's, five B's and three C's?

2. How many lattice paths from $(0, 0)$ to $(7, 7)$ do travel through any point above the diagonal?

3. How many integer valued solutions to the following equations and inequalities:

a. $x_1 + x_2 + x_3 + x_4 = 52$, all $x_i > 0$.

b. $x_1 + x_2 + x_3 + x_4 = 52$, all $x_i \geq 0$.

c. $x_1 + x_2 + x_3 + x_4 < 52$, all $x_i > 0$.

d. $x_1 + x_2 + x_3 + x_4 \leq 52$, all $x_i \geq 0$.

e. $x_1 + x_2 + x_3 + x_4 = 52$, $x_1, x_3, x_4 > 0$, $x_2 \geq 8$.

f. $x_1 + x_2 + x_3 + x_4 = 52$, $x_1, x_3, x_4 > 0$, $0 < x_2 \leq 7$.

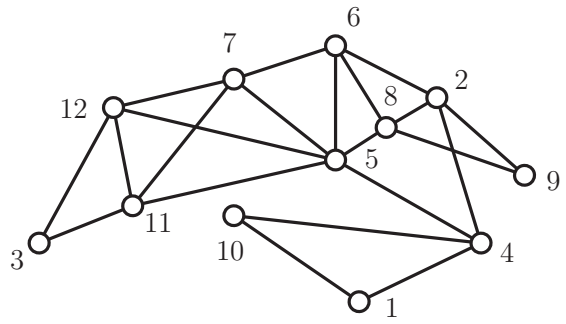
4. Find the coefficient of $a^5 b^{12} c^{21}$ in $(6a - 3b^2 - 4c^3)^{18}$

5. Use the Euclidean algorithm to find $d = \gcd(3960, 840)$.

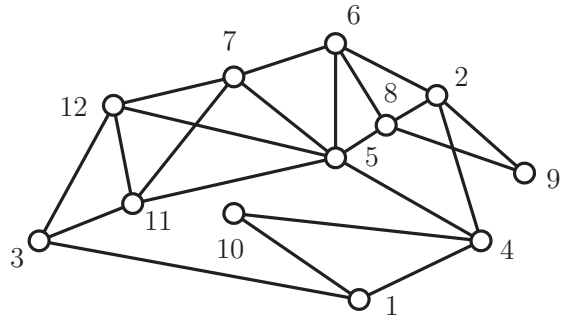
6. Use your work in the preceding problem to find integers a and b so that $d = 3960a + 840b$.

7. For a positive integer n, let t_n count the number of ternary strings of length n that do not contain 200 as a substring. Note that $t_1 = 3$, $t_2 = 9$ and $t_3 = 26$. Develop a recurrence relation for t_n and use it to compute t_4, t_5 and t_6.

8. Use the greedy algorithm developed in class (always proceed to the lowest legal vertex) to find an Euler circuit in the graph G shown below (use node 1 as root):



9. For the graph below,



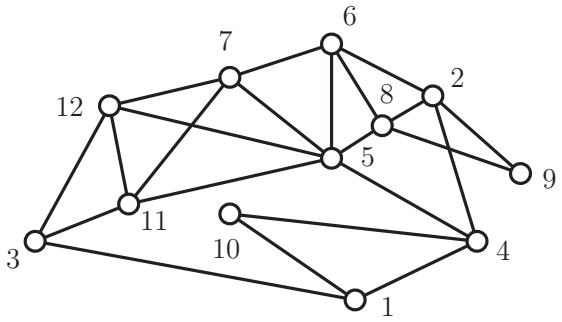
(a) Find a clique of size 4. _____

(b) Find an induced cycle of size 5. _____

(c) Show that $\chi(G) \leq 4$ by producing a proper coloring using the elements of $\{1, 2, 3, 4\}$ as colors. You may write directly on the figure.

10. Draw a diagram of a tree on 12 vertices with exactly five leaves and exactly one vertex of degree 5.

11. Show that the following graph has a hamiltonian cycle. You may either darken the appropriate edges or provide a suitable permutation of the vertex set.



12. True–False. Mark in the left margin.

1. $P(8,3) = 330$.
2. $C(8,3) = 65$.
3. If 67 pigeons are placed in 5 holes, then there is some hole with at least 13 pigeons.
4. If $f(n) = 624n^2 + 90n + 48n \log n$, and $g(n) = 3n^2 + 7n$, then $f(n) = O(g(n))$.
5. If $f(n) = 624n^2 + 90n + 48n \log n$, and $g(n) = 3n^2 + 7n$, then $g(n) = o(f(n))$.
6. $\log n = o(\sqrt{n})$, $\sqrt{n} = o(n)$, $n = o(n \log n)$, $n \log n = o(n^2)$, $n^2 = o(n^3)$ and $n^3 = o(2^n)$.
7. Any graph with 16 vertices and 153 edges has a hamiltonian cycle.