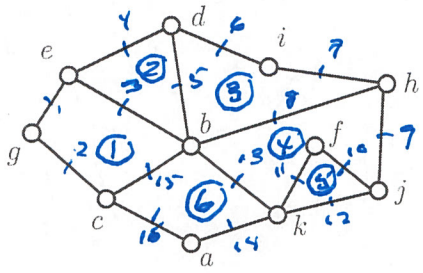
Solutions

Student Name and ID Number

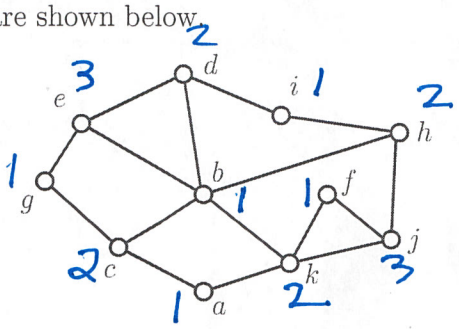
MATH 3012 Quiz 2, October 19, 2017 WTT

20 = 8+4+4+4

1. Two copies of the same planar graph G are shown below.



(7)



a. Verify Euler's formula for the graph G. You may mark on the left copy of G if you find it convenient to do so.

$V - E + F = 2$

$V = 11 \quad E = 16 \quad F = 7$

$11 - 16 + 7 = 2$ ✓

b. Find three vertices of G which form a clique of size 3. {b, e, d} or {f, j, k}

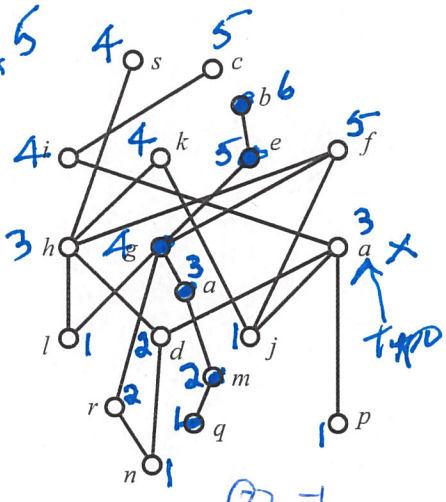
c. Show that $\chi(G) = \omega(G) = 3$ by indicating a 3-coloring of G on the right copy. A correct answer is shown. There are many others

d. Explain why G is not perfect by listing a sequence of vertices showing that G contains an induced cycle of size 9.

(a, c, g, e, d, i, h, j, k)

26 = 7+3+5

2 ~~3~~. Consider the following poset.



a. Find all points comparable to g. {a, m, q, r, n, l, b, e, f}

b. Find all points which cover g. {e, f}

c. Find all points which are covered by g. {a, r, l}

d. Find a maximal chain of size 2. {j, k} or {f, j}

e. Find a maximal chain of size 3. {f, j, l} or {a, h, l}

f. Find the set of all maximal elements. {a, c, k, b, f}

g. Find the set of all minimal elements. {l, n, q, j, p}

h. Using the algorithm taught in class (recursively removing the set of minimal elements), find the height h of the poset and a partition of P into h antichains. Also find a maximum chain. You may indicate the partition by writing directly on the diagam.

The height h is 6 and {b, e, g, a, m, q} is a maximum chain.

Page Total 46

10

10 × 1 = 10

5 6. True–False. Mark in the left margin.

T 1. There is a graph G with $\omega(G) = 2$ and $\chi(G) = 100$. Shift graph, for example

T 2. There is a graph G with $\omega(G) = 3$ and $\chi(G) = 100$. shift graph + triangle

F 3. There is a planar graph G with $\omega(G) = 2$ and $\chi(G) = 100$. 4-color theorem

T 4. If $\chi(G) = 2$, then G is perfect.

F 5. If $\chi(G) = 3$, then G is perfect. C_5

T 6. There is a graph G with 24 vertices and 100 edges such that $\chi(G) = \omega(G) = 2$.

F 7. There is a planar graph with 24 vertices and 100 edges. $E \leq 3V - 6$

F 8. There is a poset with 3209 points having width 79 and height ~~40~~ 39

T 9. There is a poset with 3209 points having width 97 and height 94.

F 10. When $n \geq 3$, the shift graph S_n contains a triangle.

12 12

100 < 144

79 · 39 < 80 · 40

= 3200

Typo

Page total 10

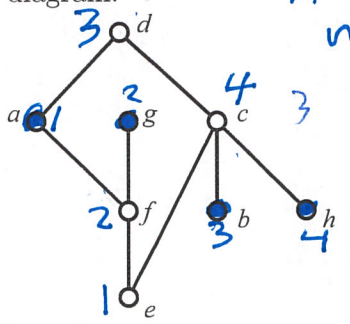
All pages: 46 + 44 + 10 = 100

3 4. Find by inspection the width w of the following poset and find a partition of the poset into w chains. Also find a maximum antichain. You may indicate the partition by writing directly on the diagram.

12

= 6+6

A correct solution is shown. There are many others.



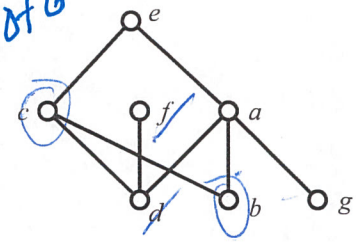
a. The width w is 4 and $\{a, g, b, h\}$ is a maximum antichain.

b. This poset is not an interval order. Find by inspection four points which form a copy of $\mathbf{2+2}$. $\{a, f, b, c\}$ or $\{a, f, h, c\}$ or $\{g, f, b, c\}$ or $\{g, f, h, c\}$

4 5. Shown below is the diagram of an interval order. Use the algorithm taught in class to find an interval representation by computing the down-sets and up-sets in the space provided. Then use the First Fit coloring algorithm to find the width w and a partition of the poset into w chains. Also, find a maximum antichain.

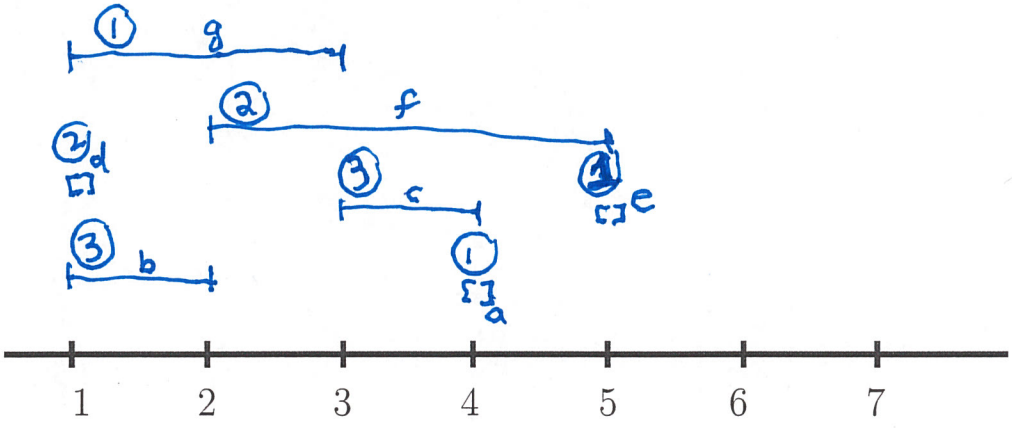
32

10+10+10+6



4 $D(a) =$ b d g
1 $D(b) =$ ∅
3 $D(c) =$ b d
1 $D(d) =$ ∅
5 $D(e) =$ a c b d g
2 $D(f) =$ d
1 $D(g) =$ ∅

$U(a) =$ e 4
$U(b) =$ a c e 2
$U(c) =$ e 4
$U(d) =$ a c e f 1
$U(e) =$ ∅ 5
$U(f) =$ ∅ 5
$U(g) =$ a e 3



The width w is 3 and $\{b, d, g\}$ is a maximum antichain.

also $\{c, f, g\}$, $\{a, c, f\}$, $\{b, f, g\}$

Page Total 44