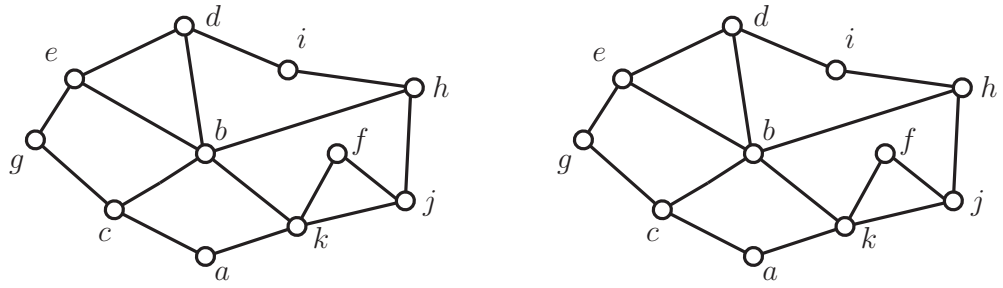
Student Name and ID Number

MATH 3012 Quiz 2, October 19, 2017 WTT

1. Two copies of the same planar graph G are shown below.



a. Verify Euler's formula for the graph G. You may mark on the left copy of G if you find it convenient to do so.

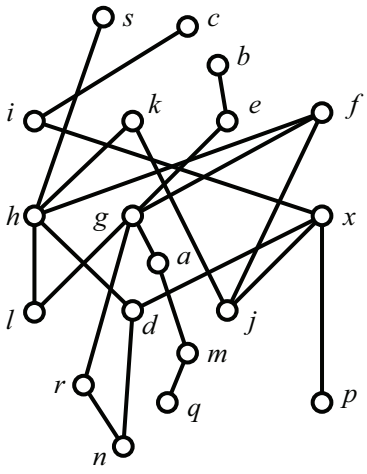
b. Find three vertices of G which form a clique of size 3. _____

c. Show that $\chi(G) = \omega(G) = 3$ by indicating a 3-coloring of G on the right copy.

d. Explain why G is not perfect by listing a sequence of vertices showing that G contains an induced cycle of size 9.

_____

2. Consider the following poset.



a. Find all points comparable to g. _____

b. Find all points which cover g. _____

c. Find all points which are covered by g. _____

d. Find a maximal chain of size 2. _____

e. Find a maximal chain of size 3. _____

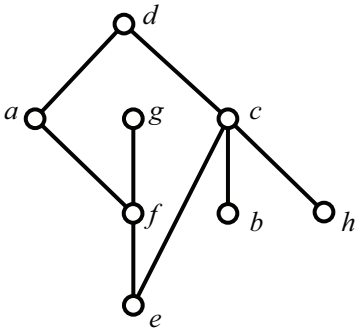
f. Find the set of all maximal elements. _____

g. Find the set of all minimal elements. _____

h. Using the algorithm taught in class (recursively removing the set of minimal elements), find the height h of the poset and a partition of P into h antichains. Also find a maximum chain. You may indicate the partition by writing directly on the diagam.

The height h is _____ and _____ is a maximum chain.

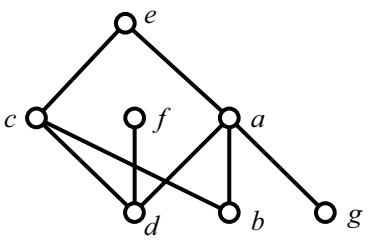
3. Find by inspection the width w of the following poset and find a partition of the poset into w chains. Also find a maximum antichain. You may indicate the partition by writing directly on the diagram.



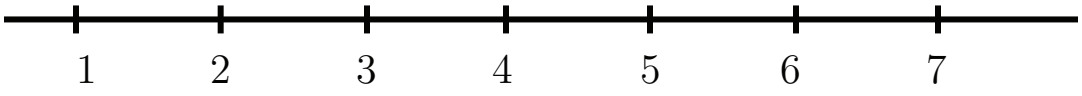
a. The width w is _____ and _____ is a maximum antichain.

b. This poset is not an interval order. Find by inspection four points which form a copy of $\mathbf{2}+\mathbf{2}$. _____.

4. Shown below is the diagram of an interval order. Use the algorithm taught in class to find an interval representation by computing the down-sets and up-sets in the space provided. Then use the First Fit coloring algorithm to find the width w and a partition of the poset into w chains. Also, find a maximum antichain.



$D(a) =$ $U(a) =$
$D(b) =$ $U(b) =$
$D(c) =$ $U(c) =$
$D(d) =$ $U(d) =$
$D(e) =$ $U(e) =$
$D(f) =$ $U(f) =$
$D(g) =$ $U(g) =$



The width w is _____ and _____ is a maximum antichain.

5. True–False. Mark in the left margin.

1. There is a graph G with $\omega(G) = 2$ and $\chi(G) = 100$.
2. There is a graph G with $\omega(G) = 3$ and $\chi(G) = 100$.
3. There is a planar graph G with $\omega(G) = 2$ and $\chi(G) = 100$.
4. If $\chi(G) = 2$, then G is perfect.
5. If $\chi(G) = 3$, then G is perfect.
6. There is a graph G with 24 vertices and 100 edges such that $\chi(G) = \omega(G) = 2$.
7. There is a planar graph with 24 vertices and 100 edges.
8. There is a poset with 3209 points having width 79 and height 39.
9. There is a poset with 3209 points having width 97 and height 93.
10. When $n \geq 3$, the shift graph S_n contains a triangle.