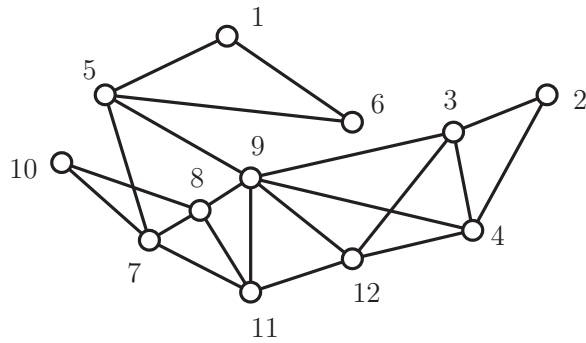


MATH 3012 Quiz 1, February 5, 2018, WTT

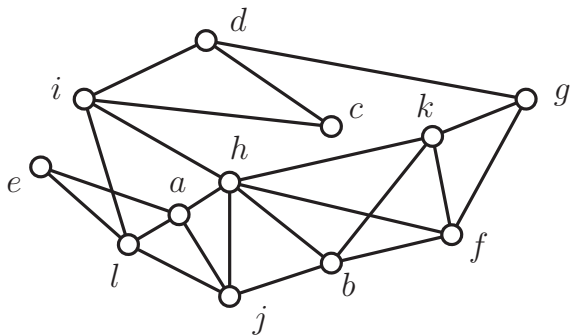
1. Consider the 52-element set consisting of all letters of the English alphabet, including both lower case and upper case.
 - a. How many strings of length 42 can be formed if repetition of symbols is *not* permitted?
 - b. How many strings of length 42 can be formed if repetition of symbols is permitted?
 - c. How many strings of length 42 can be formed using exactly seven *A*'s, twenty *a*'s, ten *B*'s and five *b*'s?
2. How many lattice paths from $(0,0)$ to $(14,14)$ do not pass through any point above the diagonal?
3. How many integer valued solutions to the following equations and inequalities:
 - a. $x_1 + x_2 + x_3 + x_4 + x_5 = 81$, all $x_i > 0$.
 - b. $x_1 + x_2 + x_3 + x_4 + x_5 = 81$, all $x_i \geq 0$.
 - c. $x_1 + x_2 + x_3 + x_4 + x_5 < 81$, all $x_i > 0$.
 - d. $x_1 + x_2 + x_3 + x_4 + x_5 = 81$, $x_1, x_3, x_4, x_5 > 0$, $7 < x_2 < 13$.
4. Find the coefficient of $x^4y^{26}z^8$ in $(7xy - 9y^2 + 5z)^{23}$

5. Use the Euclidean algorithm to find $d = \gcd(2366, 520)$.
6. Use your work in the preceding problem to find integers a and b so that $d = 2366a + 520b$.
7. For a positive integer n , let t_n count the number of ways to tile a $2 \times n$ grid with tiles of three shapes: (1) a 2×2 square, (2) a 1×2 rectangular tile and (3) a 2×1 rectangular tile. Develop initial conditions and a recurrence relation satisfied by t_n and use it to compute t_1, t_2, t_3, t_4 and t_5 .

8. Use the greedy algorithm developed in class (always proceed to the lowest legal vertex) to find an Euler circuit in the graph G shown below (use node 1 as root):



9. For the graph shown below:



- a. Find a clique of size 4. _____
- b. Find an induced cycle of size 5. _____
- c. Show that $\chi(G) \leq 4$ by producing a proper coloring using the elements of $\{1, 2, 3, 4\}$ as colors. Write directly on the figure to give your answer.
- d. Explain why this graph does not have an Eulerian circuit.
- e. Show that the graph is Hamiltonian by listing an appropriate sequence of vertices below.

10. True–False. Mark in the left margin.

1. $P(10, 4) = 720$.
2. $C(10, 4) = 210$.
3. Any connected graph with an even number of edges has an Euler circuit.
4. There is a connected graph with 500 vertices and 5000 edges which does not have a Hamiltonian cycle.
5. The number of lattice paths from $(0, 0)$ to $(12, 12)$ which pass through $(6, 8)$ is $C(12, 6)C(12, 8)$.
6. If G is a graph and $\chi(G) = 3$, then $\omega(G) = 3$.
7. The running time of an optimal implementation of the Euler Circuit algorithm is linear in the input size.
8. The decision problems: Is $\chi(G) \leq 3$? and Is G Hamiltonian? belong to the class NP .
9. The decision problems: Is $\chi(G) \leq 2$? and Is G Eulerian? belong to the class P .
10. *Just for fun and not graded.* Connected tilings of recursive multinomial graphs admit Hamiltonian circuits with vertices having clique size larger than their chromatic number, except when they have useless certificates whose correctness can be estimated in polynomial time by a biased referee.