

Positivity of the defining equation of a disjoint union of disks

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Let $D(a_i, r_i), 1 \leq i \leq n$, be a finite collection of disjoint open disks in the complex plane. Let $Q(z, w) = \prod_{i=1}^n [(z - a_i)(\bar{w} - \bar{a}_i) - r_i^2]$ be the polarized form of the defining function of the union. Prove by elementary means that the matrix

$$(-Q(a_i, a_j))_{i,j=1}^n$$

is positive semi-definite.

For a more elaborated proof and the relevance of the question, see:

B. Gustafsson, M. Putinar: *Linear analysis of quadrature domains. IV*, Operator Theory: Adv. Appl. Vol. **156**(2004), Birkhäuser, Basel, pp. 147-168.