Minimizing Multiplier polynomials

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In some investigations of Padé approximation, the following problem arises: let Q be a given polynomial of degree n, and $|z| < 1$. Find a polynomial S of degree n that minimizes

$$\Lambda_n(Q, z) = \frac{\max_{|t|=1} |QS|(t)}{|QS|(z)}$$

over all polynomials S of degree n. That is,

$$\Lambda_n(Q, z) = \inf_{\deg(S)=n} \frac{\max_{|t|=1} |QS|(t)}{|QS|(z)}$$

If

$$Q(z) = \prod_{j=1}^{n} (z - z_j),$$

then it is easy to see that

$$\Lambda_n(Q, z) \leq \prod_{j=1}^{n} \Lambda_1(\ell_j; z)$$

where for each j,

$$\ell_j(t) = t - z_j.$$

Problem:
Compute $\Lambda_1(\ell; z)$ where $\ell(t) = t - a$ is a linear function.

This looks trivial, doesn't it? Try it for a while, and you will see it is not so simple.