Integral representation of some functions related to the Gamma function

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A function $f : (0, \infty) \to (0, \infty)$ is called *logarithmically completely monotonic* if $\log f$ is completely monotonic, i.e. $(-1)^n D^n(\log f) \geq 0$ for $n = 0, 1, \ldots$. The class of these functions is denoted $\mathcal{L}$. Recently Feng, Guo and Chen [2] proved that

$$\Phi(x) = [\Gamma(x+1)]^{1/x}(1 + 1/x)^x/x \in \mathcal{L}.$$

We prove in [1] that Stieltjes transforms belong to $\mathcal{L}$ and that Φ as well as $\log \Phi$ are Stieltjes transforms. We recall that Stieltjes transforms are functions of the form

$$a + \int_0^\infty \frac{d\mu(s)}{s+x},$$

where $a \geq 0$ and μ is a non-negative measure.

[1] C. Berg, *Integral representation of some functions related to the Gamma function*, to appear in Mediterranean J. Math.

[2] Feng Qi, Bai-Ni Guo, and Chao-Ping Chen, *Some completely monotonic functions involving the Gamma and polygamma functions* RGMIA Res. Rep. Coll. **7**, No. 1 (2004), Art. 8. Available online at http://rgmia.vu.edu.au/v7n1.html