

**Minimal Riesz energy point configurations for
rectifiable d -dimensional manifolds**

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We investigate the energy of arrangements of N points on a rectifiable d -dimensional manifold $A \subset \mathbf{R}^{d'}$ that interact through the power law (Riesz) potential $V = 1/r^s$, where $s > 0$ and r is Euclidean distance in $\mathbf{R}^{d'}$. With $\mathcal{E}(\mathcal{A}, \mathcal{N})$ denoting the *minimal* energy for such N -point configurations, we determine the asymptotic behavior (as $N \rightarrow \infty$) of $\mathcal{E}(\mathcal{A}, \mathcal{N})$ for each fixed $s \geq d$. Moreover, if A has positive d -dimensional Hausdorff measure, we show that N -point configurations on A that minimize the s -energy are asymptotically uniformly distributed with respect to d -dimensional Hausdorff measure on A when $s \geq d$. Even for the unit sphere $S^d \subset \mathbf{R}^{d+1}$, these results are new.