Cubature on the Sphere – Optimal Estimates of the Worst-Case Error in Sobolev Spaces

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In this talk I will present results, from joint work with Ian H. Sloan, on the rate of convergence of cubature (numerical integration) rules on the sphere S^2. We consider sequences (Q_n) of cubature rules Q_n with the following properties: (i) Q_n is exact for spherical polynomials of all degrees up to n, and (ii) Q_n has positive weights (or alternatively the sequence (Q_n) satisfies a certain regularity property). We show with a novel argument, that the worst-case cubature error in the Sobolev space $H^s(S^2)$, $s > 1$, for such a sequence of cubature rules has the optimal order of convergence $O(n^{-s})$. Examples of rules with the required properties include positive weight product Gauss rules and the equal weight rules based on any spherical n-design. Interpolatory cubature rules based on extremal fundamental systems also satisfy these assumptions if they turn out to have positive weights (as is indicated by numerical evidence).