Zeroes of orthogonal polynomials, best approximation, and constructive solutions to inverse sources problems

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We consider some practical inverse problems for Laplace operator that consist in recovering pointwise sources distributed in a 2D or 3D domain from available measurements of the solution on the boundary. In the electrical framework for example, and in medicalengineering, this comes with the inverse EEG (electroencephalography) problem that consists in detecting pointwise dipolar current sources (epileptic foci) located in the brain from measures of the electrical difference of potentials on the scalp.

These are inverse potential problems (Newton or Green) that we preliminary approach in balls. In the 2D setting where the Newtonian and the logarithmic potential coincide, harmonic conjugation provides us with the values on the circle of the Cauchy integral of a measure supported on a curve joining the sources inside the disk. In 3D, the problem can still be efficiently expressed on families of 2D disks consisting in planar cross sections of the domain. Some computations are needed in order to show that available data on their boundaries still coincide with those of some Cauchy integrals in the associated disks.

This allows in both cases to express the denominators of best uniform or quadratic meromorphic approximants (with prescribed degree and poles in the disk) to such a function on the circle as orthogonal polynomials with respect to some complex measure linked to the original one. Their zeroes are then the poles of the approximants, and possess nice convergence properties towards the sources (singularities) to be recovered (talk by L. Baratchart). The computation of such approximants on the boundary thus leads to constructive resolution schemes for this class of inverse problems. This is strongly related to a previous joint work with Edward B. Saff in which we exhibited such links for other non-destructive control issues such as cracks detection.

I will precise these inverse problems in the 2D and 3D settings and their links with best meropmorphic approximation in the complex plane. Numerical computations will be discussed, and further related issues.