Ratio asymptotics for multi-orthogonal polynomials of Nikishin systems

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Let $S = (s_1, \ldots, s_m)$ be a Nikishin system of measures supported on an interval $[a, b]$ of the real line such that $s'_k > 0, k = 1, \ldots, m,$ a.e. on $[a, b]$. It is known that there exists a unique monic polynomial Q_n of degree nm such that for each $k \in \{1, \ldots, m\}$ and $j = 0, \ldots, n-1$.

$$\int x^j Q_n(x) ds_k(x) = 0\,.$$

We prove that there exists a function Φ holomorphic in the complement D of $[a, b]$ in the complex plane such that

$$\lim_{n\to\infty} \frac{Q_{n+1}}{Q_{\mathbf{n}}} = \phi$$

uniformly on compact subsets of D. This result extends to this setting E. A. Rakhmanov's Theorem on ratio asymptotics of orthogonal polynomials on the real line.