Which Weights on R Admit Jackson Theorems?

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Let $W : \mathbb{R} \to (0, \infty)$ be continuous. Does W admit a Jackson or Jackson-Favard Inequality? That is, does there exist a sequence $\{\eta_n\}_{n=1}^{\infty}$ of positive numbers with limit 0 such that for $1 \leq p \leq \infty$,

$$\inf_{\deg(P) \leq n} \| (f - P)W \|_{L_p(\mathbb{R})} \leq \eta_n \| f'W \|_{L_p(\mathbb{R})}$$

for all absolutely continuous f with $\| f'W \|_{L_p(\mathbb{R})}$ finite? We show that such a theorem is true iff both

$$\lim_{x \to \infty} W(x) \int_0^x W^{-1} = 0$$

and

$$\lim_{x \to \infty} \left(\sup_{[0,x]} W^{-1} \right) \int_x^{\infty} W = 0,$$

with analogous limits as $x \to -\infty$. In particular $W(x) = \exp(-|x|)$ does not admit a Jackson theorem, although it is well known that $W(x) = \exp(-|x|^{\alpha}), \alpha > 1$, does. We also construct weights that admit an L_1 but not an L_∞ Jackson theorem (or conversely).

The talk will be introductory, and might be accessible to those to whom Jackson and Bernstein sound like the directors of a large corporation.