On Kaczmarz's method for solving infinite systems of linear equations in infinite dimensional spaces

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Abstract

Given a sequence of unit vectors $a_k, k = 1, ..., m$ in R^d, for each sequence of numbers $\alpha_k, k = 1, .., m$ the solution (if it exists) to the system of linear equations $< a_k, x >= \alpha_k,\ k = 1, ..., m$ can be obtained as the limit of the sequence $x_n, n = 0, 1, ...$ defined by the recurence: $x_0 = 0, x_n = x_{n-1} + (\alpha_n - < a_n, x_{n-1} >) a_n$ where the sequences (a_n) and (α_n) are extended to infinite sequences by making them m-periodic.

Motivated by a problem from Learning Theory we consider infinite sequences (a_k) in Hilbert spaces for which the above recurence leads to a solution whenever it exists. We prove that "most" of the sequences (a_k) have this property. This is obtained as a consequence of a result on the almost sure convergence of compositions of identically distributed random projections in Hilbert space. Also using complex inner functions we give a characterization of stationary sequences in Hilbert space with the above property. Both results have their interpretation in learning theory.