

## SEAM 2018, ABSTRACTS

### **Meric Augat (University of Florida); The free Grothendieck theorem.**

A remarkable pair of theorems of Grothendieck say if  $p : C^g \rightarrow C^g$  is an injective polynomial, then  $p$  is bijective and its inverse is a polynomial. We prove a free analog of this. A free polynomial mapping in  $g$  freely non-commuting variables sends  $g$ -tuples of matrices (of the same size) to  $g$ -tuples of matrices (of the same size).

Our result is as follows; if  $p$  is a free polynomial mapping that is injective, then it has a free polynomial inverse. We will make use of a free version of the Jacobian Conjecture as well as results from free analysis, formal power series and skew fields. The Free Grothendieck Theorem is related to free analysis, automorphisms of the free algebra and tame vs. wild automorphism of the free algebra.

### **Debendra P. Banjade (Coastal Carolina University); On a generalization of Wolff's Ideal Theorem for certain subalgebras of $H^\infty(\mathbb{D})$ .**

We prove the generalized Wolff's Ideal Theorem on certain uniformly closed subalgebras of  $H^\infty(\mathbb{D})$  on which the Corona Theorem is already known to hold. Moreover, we provide a counter example for the case when  $F(0) = 0$ .

### **Dmitriy Bilyk (University of Minnesota); Point distributions on the sphere: Energy minimization, discrepancy, and more.**

The quality of a finite point distribution may be measured in many different ways depending on the problem at hand: discrepancy, numerical integration, various energies, packing, etc. There are numerous connections between these topics, and we shall explore some of the lesser known ones, in particular, different versions of Stolarsky principle, which relates discrepancy on the sphere to energy minimization. We shall also discuss applications of these topics to problems in other fields (signal processing, discrete and combinatorial geometry), which can be formulated as energy minimization problems.

### **Vladimir Bolotnikov (College of William and Mary); Boundedness of commutators on weighted Hardy spaces.**

Let  $\mathcal{P}$  denote the Pick class of analytic self-mappings of the upper half plane. The Stieltjes class  $\mathcal{S}$  can be defined as the class of Pick functions  $f(z)$  such that  $zf(z)$

is also in  $\mathcal{P}$ . Stieltjes functions turn out to be analytic on  $\mathbb{R}_- = (-\infty, 0)$  and their restrictions to  $\mathbb{R}_-$  are characterized as nonnegative operator monotone functions on  $\mathbb{R}_-$ . The Carathéodory-Fejér problem  $\mathcal{CF}_n(S, x_0)$  consists of finding a function  $f \in \mathcal{S}$  with prescribed  $f^{(j)}(x_0) = c_j$  ( $j = 0, \dots, n-1$ ) if  $x_0 \in \mathbb{C} \setminus \mathbb{R}_-$ . If  $x_0 \geq 0$ , then by  $f^{(j)}(x_0)$  we mean the limit of  $f^{(j)}(z)$  as  $z$  tends to  $x_0$  nontangentially.

**Grant Boquet (Lawrence Livermore National Laboratory).** In this talk we present conditions for two-dimensional behavioral systems to have a state space representation. We begin with a brief overview of the behavioral approach to  $n$ -dimensional systems theory and highlight key results that establish a correspondence between algebraic conditions of a system's kernel representation and properties of its signal space (e.g. controllability). We conclude with necessary and sufficient conditions for a two-dimensional behavior to have a Fornasini-Marchesini state space representation or, when the system is autonomous, a Livsic state space representation.

**Umut Calgar, Florida International University; Isoperimetric and information theoretic inequalities for log concave functions.** For log concave functions, we introduce linear invariant divergence inequalities originating in an information-theoretic inequality. This leads to new (and some existing) inequalities for functional and geometric  $L_p$ -affine surface areas. This is a joint work with E. Werner.

**Raymond Cheng (Old Dominion University); On the function theory of  $\ell_A^p$ .** For  $1 < p < \infty$ , Let  $\ell_A^p$  be the space of analytic functions on the unit disk  $D$  such that their Taylor coefficients lie in the sequence space  $\ell^p$ .

We will review what we little know about  $\ell_A^p$  compared to the Hardy space  $H^2$ . Then we will look at some ideas from Banach space geometry that might offer a path forward. Some results derived using those tools will be presented, concerning zero sets, a possible inner-outer factorization, and shift-invariant subspaces.

**Cheng Chu (Vanderbilt University); Sub-Bergman Hilbert spaces.** For an analytic function on the unit disk bounded by 1, let  $T_b$  be the Toeplitz operator on the Bergman space  $A_2$ . The range of the operator  $(I \otimes T_b T_b^*)^{1/2}$  is denoted by  $A(b)$ , and is endowed with the Hilbert space structure. These spaces are analogues of de Branges-Rovnyak spaces in the Bergman space setting and are called sub-Bergman Hilbert spaces. Kehe Zhu proved that if  $b$  is a finite Blaschke product

then the space  $A(b)$  is the Hardy space. In this talk, we will discuss an alternative approach to Zhu's theorem and obtain a stronger result. In addition, we show that the polynomials are dense in sub-Bergman Hilbert spaces.

**Marianna Csornei (University of Chicago); The Kakeya needle problem for rectifiable sets.** We show that the classical results about rotating a line segment in arbitrarily small area, and the existence of a Besicovitch and a Nikodym set hold if we replace the line segment by an arbitrary rectifiable set. This is a joint work with Alan Chang.

**Stephen Deterding (University of Kentucky); Bounded point derivations on certain function spaces.** Let  $X$  be a compact subset of the complex plane. We denote by  $R(X)$  the uniform closure of rational functions with poles off  $X$  and by  $R^p(X)$  the  $L^p$  closure of rational functions with poles off  $X$ . Bounded point derivations on  $R(X)$  or  $R^p(X)$  are bounded linear functional that are studied to determine how much differentiability is preserved under convergence in either the uniform or  $L^p$  norms. In this talk we consider the relationship between bounded point derivations and the usual notion of the derivative to see how close functions in  $R(X)$  or  $R^p(X)$  come to being differentiable.

**Francesco Di Plinio (University of Virginia); On the Hilbert transform along vector fields in higher dimensions.** I will present the recent proof of the sharp estimate, in terms of cardinality and of admissible weights for the Hilbert transform along a finite order three dimensional lacunary set of directions. I will discuss the connection with the Hilbert transform along Lipschitz vector field in dimension three and higher. Joint work with Ioannis Parisis (UPV-EHU)

**Chris Discenza (University of Florida); Resonance clusters and wave turbulence regimes of edgewaves.** The evolution of edgewaves in the nearshore provides an interesting example of a weakly nonlinear system of three wave interactions with a discrete spectrum. We consider the edgewave problem under reasonably simple assumptions that has a known homogeneous dispersion relation with discrete modes. We construct a topological model for the resonance cluster which allows us to form some general conclusions about the dynamic cascade. We then formulate and describe the quasi resonant conditions that allow for wave kinetics.

**Yen Do (University of Virginia); Real roots of random algebraic polynomials.** A random algebraic polynomial is a polynomial with random coefficients, and estimating the number of real roots for these polynomials is an important question with a long history. We discuss some recent progress in this direction, in particular we will discuss some ideas in a recent joint work with V. Vu where we prove a central limit theorem for the number of real roots for random Weyl polynomials.

**Quanlei Fang (University of Alabama); Multipliers of Drury-Arveson space.** The Drury-Arveson Space, as a Hilbert function space, plays an important role in multivariable operator theory. In this talk we will discuss various properties of multipliers (particularly the Schur class multipliers) of the Drury-Arveson space.

**Tim Ferguson, (University of Alabama); The range and valence of a real Smirnov function.** We give a complete description of the possible ranges of real Smirnov functions (quotients of two bounded analytic functions on the open unit disk where the denominator is outer and such that the radial boundary values are real almost everywhere on the unit circle). Our techniques use the theory of unbounded symmetric Toeplitz operators, some general theory of unbounded symmetric operators, classical Hardy spaces, and an application of the uniformization theorem. In addition, we completely characterize the possible valences for these real Smirnov functions when the valence is finite. To do so we construct Riemann surfaces we call disk trees by welding together copies of the unit disk and its complement in the Riemann sphere. We also make use of certain trees we call valence trees that mirror the structure of disk trees. Joint work with Bill Ross.

**Matt Fleeman (Baylor University); Hyponormal Toeplitz operators with non-harmonic symbol acting on the Bergman space.** The Toeplitz operator acting on the Bergman space  $A_2(D)$ , with symbol  $\Phi$  is given by  $T\Phi f = P(\Phi f)$ , where  $P$  is the projection from  $L_2(D)$  onto the Bergman space. We present some history on the study of hyponormal Toeplitz operators acting on  $A_2(D)$ , as well as give results for when  $f$  is a non-harmonic polynomial. Particular attention is given to unusual hyponormality behavior that arises due to the extension of the class of allowed symbols.

**Aaron Ernesto Ramirez Flores (University of El Salvador); Asymptotic results of localized Toeplitz operators.** Given a Hilbert space with a continuous frame satisfying certain

localization conditions, we derive various asymptotic results of compact and trace-class Toeplitz operators defined with respect the continuous frame. Some important applications will be mentioned, such as results about the distribution of eigenvalues of these Toeplitz operators.

**Walton Green (Clemson University); Observability of a Visco-Elastic wave equation.**

We give a proof of the Neumann boundary observability inequality for the visco-elastic wave equation (also called wave equation with memory kernel) in an arbitrary space dimension. We argue by perturbation from the standard wave equation to show that the corresponding harmonic system is a Riesz sequence. Joint work with Mishko Mitkovski and Shitao Liu.

**Erin Griesenauer (Eckerd college); Matrix bundles and function algebras.** We study algebras of cross sections of holomorphic matrix bundles. Our work is inspired by certain matrix bundles that arise naturally in geometric invariant theory and noncommutative function theory. We view these algebras of cross sections as noncommutative analogues of function algebras. We describe in detail how these algebras arise and state a few of their properties.

**Caixing Gu (California Polytechnic State University at San Luis Obispo); Reducibility of the power of a  $C_0(1)$  operator.** Inspired by the work of Douglas and Foias in 2006 on the structure of the square of a  $C_0(1)$  operator, we form a conjecture about a certain reducibility of any power  $N$  of a  $C_0(1)$  operator. We then prove this conjecture for  $N = 3$  by determining explicitly the relevant reducing subspaces.

**Keaton Hamm (Vanderbilt); Convergence of rearranged Fourier series.** This talk will investigate a conjecture of Ulyanov which asks if, for any continuous function, there exists a rearrangement (i.e. bijection from  $\mathbb{Z}$  to  $\mathbb{Z}$ ) such that the rearranged partial sums of the Fourier series converges uniformly to the function. We give some equivalences of this conjecture in terms of convergence of related multiplication operators in the strong or weak operator topology, and some interesting partial results related to Ulyanov's conjecture.

**Joshua Isralowitz (University of Albany, SUNY); A vector valued Fefferman-Phong inequality.** Schrödinger operators  $-\Delta + V$  with nonnegative potentials  $V$  in an appropriate reverse Hölder class  $B_p$  were first studied by Z. Shen in 1995, who later in 1999 proved optimal decay bounds for the fundamental solution of these operators. In a recent paper by B.

Davey, J. Hill, and S. Mayboroda, the authors raised the possibility of proving such estimates for these operators (and more generally uniformly elliptic systems) acting on vector valued functions with a matrix  $B_p$  potential.

In this talk, we discuss what the appropriate definition of a matrix  $B_p$  weight even is, and discuss the matrix weighted Fefferman-Phong inequality, which (as in the scalar case) plays a pivotal role in the study of these systems. This is joint work with B. Davey and S. Mayboroda.

**Benjamin Jaye (Clemson University); Reflectionless measures for singular integral operators.** In this talk we consider the problem of understanding the geometric properties of a measure that are inherited from the boundedness in  $L^2$  of an associated singular integral operator. We shall show how to reduce several questions in this area to understanding a certain limiting object called a reflectionless measure. Several open problems will be discussed. Joint work with Fedor Nazarov.

**Marie-Jose Kuffner (University of Washington at Saint-Louis); Boundedness of commutators on weighted Hardy spaces.** It is known that boundedness of the commutator  $[b, H]$  on weighted  $L^p$ -spaces for  $1 < p < \infty$  is characterized by  $b$  being in a certain  $BMO$  space adapted to the given weights. In this talk, we present the case  $p = 1$  and discuss the space that characterizes boundedness of  $[b, H]$  on the weighted Hardy space  $H^1(w)$  for certain  $A_p$  weights.

**Hyun Kwon, University of Alabama; Similarity and reducing subspaces of the multiplication operator on the bidisk.** We consider the weighted Bergman space of the bidisk and answer some questions about a multiplication operator defined on it. This talk is based on joint work with Hui Li and Yucheng Li.

**Michael Lacey (Georgia Tech); Sparse bounds: Recent results.** Sparse bounds provide a quantification of (weak) type bounds for operators. They immediately give fully quantitative weighted estimates in the category of Muckenhoupt and Reverse Holder classes. They also are broadly applicable. We will give a few recent results.

**Marc Mancuso (Louisiana State); Noncommutative function theory on operator domains.** We discuss noncommutative (nc) functions on domains of operators on an infinite dimensional Hilbert space. The main results are (global) inverse and implicit function theorems for nc

functions defined on a connected domain. The hypotheses of these theorems involve conditions on the derivative of the function, and we will show how to relax these conditions in the case where the function is suitably continuous in the strong operator topology.

**Ryan Matzke (University of Minnesota); On Fejes Tóth's conjectures on the sums of angles.** In 1959, Fejes Tóth posed two conjectures about optimal point distributions on the sphere, more precisely, about the maximizers of the following discrete energies: (a) the pairwise sum of angles (geodesic distances) between  $N$  unit vectors in  $\mathbb{S}^d$ , (b) the sum of non-obtuse angles, i.e. the sum of angles between  $N$  lines defined by vectors in  $\mathbb{S}^d$ . Continuous versions of these conjectures can also be stated. While the questions are clearly related, their nature is rather different: the first case induces strong "repulsion", while the second imposes orthogonality. Through a new analogue of the Stolarsky Invariance Principle from discrepancy theory, we characterize the maximizers of the sum of angles, verifying the first conjecture. We also provide improved energy bounds for the sum of line angles by relating this problem to the so-called frame potential. In addition, we discuss several new solutions to the only settled case of the second conjecture  $d = 1$ .

**Scott Mccullough (University of Florida); Free convex sets.** The talk will cover some recent on free (synonymously, matrix) convex sets.

**Tomas Merchant (Kent State University); Singular integrals and symmetric measures.** In the talk we will discuss the relation between the existence of the principal value integral for some Caldern-Zygmund operators and properties involving the transportation coefficients with respect to the symmetric measures generated by the kernel. In addition, we will be able to classify those symmetric measures in some cases.

**Mishko Mitkowski (Clemson University); Compactness and frames.** I will present a characterization for compactness of bounded linear operators in terms of their behavior on certain classes of frames. I will show how this result can be used to derive compactness criteria for several classes of operators including Toeplitz operators on various function spaces, some singular integral operators, and some pseudodifferential operators.

**Michael Northington (Georgia Tech); Fourier multipliers and the strength of exponential bases.** Many questions in time-frequency analysis can be reduced to properties of a sequence of complex exponentials in certain function spaces. In this talk, we discuss a connection between both Gabor systems and shift-invariant spaces and the integer frequency exponentials,  $E = \{e^{2\pi i k \cdot x}\}_{k \in \mathbb{Z}^d}$  in the weighted space,  $L^2_W(\mathbb{T}^d)$ . Next, we show that certain basis properties of  $E$  correspond to a Fourier multiplier property related to the weight  $W$ . We then study properties which prevent such a function from being a bounded Fourier multiplier and use this to prove Balian-Low type uncertainty principles. (Joint work with Shahaf Nitzan and Alex Povel).

**Yumeng Ou (MIT); Weighted restriction estimates and Falconer's distance set problem.** In this talk, we will discuss a very recent improvement towards Falconer's distance set conjecture, i.e. any compact set in  $\mathbb{R}^d$  with Hausdorff dimension larger than  $d/2$  has distance set of strictly positive Lebesgue measure. We improve the previously best known partial results of this problem in three and higher dimensions by obtaining an improved bound of the Fourier extension operator with respect to fractal measures. The main ingredients in our proof are the method of polynomial partitioning and refined Strichartz estimates. This is joint work with X. Du, L. Guth, H. Wang, B. Wilson, and R. Zhang.

**Eyvi Palsson (Virginia Tech); Falconer type theorems for higher order point configurations.** Finding and understanding patterns in data sets is of significant importance in many applications. One example of a simple pattern is the distance between data points, which can be thought of as a 2-point configuration. Two classic questions, the Erdos distinct distance problem, which asks about the least number of distinct distances determined by  $N$  points in the plane, and its continuous analog, the Falconer distance problem, explore that simple pattern. Questions similar to the Erdos distinct distance problem and the Falconer distance problem can also be posed for more complicated patterns such as patterns based off of three points, which can be viewed as 3-point configurations. In this talk I will briefly explore such generalizations. The main techniques used come from analysis and geometric measure theory.

**Bae Jun Park (UW Madison); Some maximal inequalities on Triebel-Lizorkin spaces for  $p = \infty$ .** Fefferman-Stein's vector-valued maximal inequality plays a significant role in the theory of function spaces, especially in Triebel-Lizorkin spaces  $\dot{F}_p^{s,q}$ ,  $0 < p < \infty$ . However, the inequality cannot be applied to  $\dot{F}_\infty^{s,q}$ . We introduce a new maximal inequality that can be readily

used for  $\dot{F}_\infty^{s,q}$ , which is a “ $\dot{F}_\infty^{s,q}$ -variant” of Fefferman-Stein vector-valued inequality. We also discuss some applications of our results.

**Josiah Park (Georgia Tech); Finite Balian-Low theorems in  $\mathbb{R}^d$  and extremal Gabor frames.** We study versions of the Balian-Low theorem for finite signals in  $\mathbb{R}^d$ ,  $d \geq 2$ . Our results are generalizations of S. Nitzan and J.-F. Olsen’s recent work and show that a quantity closely related to the Balian-Low Theorem has the same asymptotic growth rate,  $O(\log N)$  for each dimension  $d$ . We discuss properties of Gabor Riesz bases corresponding to Zak-transforms minimizing a parameter  $\beta_{A,B}(N)$  that measures time-frequency localization of a generator and numerical experiments. Joint work with M. Northington.

**Bhupendra Paudyal (Central State University); The lattices of invariant subspaces of a class of operators on the Hardy space.** This work is an extension of our previous work and it describes the lattice of invariant subspaces of the shift plus a positive integer multiple of the complex Volterra operator on the Hardy space. It is motivated by a paper by Ong who studied the real version of the same operator.

**Douglas Pfeffer (University of Florida); Widom Theorem for a constrained subalgebra of  $H^\infty$ .** Given  $\phi \in L^\infty$ , the classical Widom theorem says that the associated Toeplitz operator  $T_\phi$  is left invertible if and only if  $\phi$  is norm-distance less than one from  $H^\infty$ . In this talk we consider Widom theorems for constrained subalgebras of  $H^\infty(\mathbb{D})$ . In particular we look at the algebra of bounded analytic functions on the unit disc consisting of those functions that agree at two points  $a$  and  $b$ , and establish a Widom theorem for this algebra. The proof of the classic Widom theorem considers a single Hilbert space carrying a representation for the algebra  $H^\infty(\mathbb{D})$ , namely,  $H^2$ . In the case of the constrained subalgebra, it turns out one must consider an entire family of Hilbert spaces  $H_t^2 = \{f \in H^2 \mid f(a) = tf(b)\}$ , where  $t \in C^{2,+}(\mathbb{S}^{n-1}) \cup \{\infty\}$ , and their associated Toeplitz operators  $T_\phi^t$ .

**Gabriel Prajitura (Brockport); The Berezin transform as an operator.** We introduce a type of integral operators associated with a positive measure and resembling the Berezin transforms on the unit ball. Boundedness and compactness of these Berezin type operators between weighted Bergman spaces are characterized using Carleson measures. These results are related to similar

results about Toeplitz operators between weighted Bergman spaces. This is a joint work with Ruhan Zhao and Lifang Zhou.

**Rajita Ranasinghe (University of Central Florida); An Application of the Askey-Wilson operator: Overconvergence.** In 1985, Richard Askey and James Wilson introduced the Askey-Wilson operator,  $D_q$ ,  $0 < q < 1$ , in their study of a class of orthogonal polynomials called the Askey-Wilson polynomials. It is a degree lowering operator that can be considered as a discrete version of the ordinary derivative operator. The definition of  $D_q$  presented by Askey and Wilson uses values of a function at points in the complex plane outside  $[-1, 1]$ . To make it applicable to more general classes of functions than the space of polynomials, Brown and Ismail defined  $D_q$  on a subset of the so-called  $q$ -differentiable functions of the  $L^2$ -space on  $[-1, 1]$  with respect to the weight  $(1 - x^2)^{-1/2}$ . A natural question to ask is: How smooth must a function be to ensure that it is  $q$ -differentiable? In this connection, we give a precise description of all functions that are  $q$ -differentiable in terms of the analytic continuation of the functions.

**Alexander Reznikov (Florida State); Separation and covering properties of optimal configuration for discrete Riesz potentials.** We survey recent results on the separation and covering properties for minimal Riesz energy and maximal Riesz polarization configurations and discuss several possible extensions of these results.

**Stephan Richter (University of Tennessee); Function theory for the Drury Arveson space.** The Drury-Arveson space  $H_d^2$  is the space of analytic functions in the unit ball of  $\mathbb{C}^d$  defined by the reproducing kernel  $k_w(z) = \frac{1}{1 - \langle z, w \rangle}$ . Alternately an analytic function  $f$  is in  $H_d^2$  if  $R^N f \in L^2((1 - |z|^2)^{2N-d} dV)$  for some (and hence all)  $N > (d - 1)/2$ , where  $R$  denotes the radial derivative operator  $R = \sum_{i=1}^d z_i \frac{\partial}{\partial z_i}$ .

The space has been shown to be of importance for the theory of tuples of commuting Hilbert space operators. The emerging function theory for  $H_d^2$  takes advantage of the facts that the reproducing kernel has the Pick property and that the space is a weighted Besov space.

In this talk I will speak about joint work with Aleman, Hartz, McCarthy, Perfekt, and Sunkes on multipliers and cyclic vectors for  $H_d^2$ .

**Boris Rubin (Louisiana State); Radon transforms over lower-dimensional horospheres.** We study horospherical Radon transforms that integrate functions on the  $n$ -dimensional real

hyperbolic space over horospheres of arbitrary fixed dimension  $0 < d < n$ . Exact existence conditions and new explicit inversion formulas are obtained for these transforms acting on smooth functions and functions belonging to  $L^p$ . The case  $d = n - 1$  agrees with the well-known Gelfand-Graev transform. (Joint with W. O. Bray).

**Haripada Sau, Virginia Tech); On Andô dilations for a pair of commuting contractions - Two explicit constructions and functional models.** One of the most influential results in operator theory is a result by Sz.-Nagy (1953) that states: Every contraction linear operator acting on a Hilbert space dilates to a unitary operator. After a decade, this classical dilation theory was extended elegantly by T. Andô (1963) to pairs of commuting contractions acting on a Hilbert space. However, Andô's enigmatic construction does not give any function theoretic interpretation. On the other hand, there are two concrete constructions of a unitary dilation for a single contraction by Schäffer (1955) and Douglas (1968). In this talk, a two-variable analogues of these explicit constructions will be presented to construct Andô dilation. We shall also discuss about their uniqueness. This is one of the subjects of the recent paper <https://arxiv.org/pdf/1710.11368.pdf>.

**Adisak Seesanea (University of Missouri Columbia); Regularity of positive solutions to nonlinear elliptic equations.** We present the existence and uniqueness results for positive weak solutions in certain Sobolev spaces, to nonlinear elliptic equations involving measures, with various type of operators including, the  $p$ -Laplacian, or more generally,  $\mathcal{A}$ -Laplacian, as well as the fractional Laplacian. This talk is based on joint work with Igor E. Verbitsky.

**Cody Stockdale (Washington University in St. Louis); An endpoint weak type estimate for multilinear Calderón-Zygmund operators.** It is well-known that Calderón-Zygmund operators are weak type bounded from  $L^1(\mathbb{R}^n)$  to  $L^1(\mathbb{R}^n)$ . In 2002, Grafakos and Torres prove an analogous endpoint weak type estimate for multilinear Calderón-Zygmund operators from  $(L^1(\mathbb{R}^n))^m$  to  $L^{1/m}(\mathbb{R}^n)$ . This proof utilizes the Calderón-Zygmund decomposition and follows the structure of the classical proof of the linear case. We provide an alternate proof of this weak type estimate for multilinear Calderón-Zygmund operators based on an idea of Nazarov, Treil, and Volberg. This proof avoids the use of the Calderón-Zygmund decomposition.

**Krystal Taylor (Ohio State University); A circle of problems in harmonic analysis related to curve packings.** A result due to Marstrand states that if  $r(x)$  is defined for every  $x \in \mathbb{R}^d$ , then the corresponding union of  $(d - 1)$ -dimensional spheres has positive measure. This turns out to be an application of the Stein spherical maximal theorem, although Marstrand's original proof did not rely on this result. Wolff later strengthened Marstrand's result in the plane to show that if

$$E \subset \{(x, t) : x \in \mathbb{R}^2, t > 0\}$$

with  $\dim_{\mathcal{H}}(E) > 1$ , then the corresponding union of circles  $\bigcup_{(x,t) \in E} S(x, t)$  has positive Lebesgue measure.

In a joint work with Karoly Simon, we show that if  $A$  is a sufficiently regular subset of the plane, then  $A + S^1$  has non-empty interior where  $S^1$  denotes the circle in the plane. We also determine the measure of  $A + S^1$  in the critical case when  $\dim_{\mathcal{H}}(A) = 1$ . For the question of measure, we utilize a generalized projection scheme. For the question of interior, our method relies on creating a strengthened version of a result of Erdős and Oxtoby. They show that if  $H$  is a sufficiently nice, real-valued function on the plane, and if  $A$  and  $B$  are linear sets of positive Lebesgue measure, then  $H(A, B) = \{H(x, y) : x \in A, y \in B\}$  contains a non-empty open interval.

We touch on the interplay of these problems with the study of distance sets and the existence of finite configurations within fractal sets.

**Edward Timko (University of Manitoba); A classification of  $n$ -tuples of commuting shifts of finite multiplicity.** Let  $V$  denote an  $n$ -tuple of shifts of finite multiplicity, and denote by  $\text{Ann}(V)$  the ideal consisting of polynomials  $p$  in  $n$  complex variables such that  $p(V) = 0$ . If  $W$  on  $K$  is another  $n$ -tuple of shifts of finite multiplicity, and there is a  $W$ -invariant subspace  $K'$  of finite codimension in  $K$  so that  $W|_{K'}$  is similar to  $V$ , then we write  $V \lesssim W$ . If  $W \lesssim V$  as well, then we write  $W \approx V$ .

In the case that  $\text{Ann}(V)$  is a prime ideal we show that the equivalence class of  $V$  is determined by  $\text{Ann}(V)$  and a positive integer  $k$ . More generally, the equivalence class of  $V$  is determined by  $\text{Ann}(V)$  and an  $m$ -tuple of positive integers, where  $m$  is the number of irreducible components of the zero set of  $\text{Ann}(V)$ .

**Paco Villaroja (UGA); A local  $Tb$  Theorem for compact singular integral operators with non-homogeneous measures** We introduce a new local  $Tb$  Theorem for Calderón-Zygmund operators

$$Tf(x) = \int f(t)K(t, x)d\mu(t),$$

with  $x$  not in the support of  $f$ , that extend compactly on  $L^p(\mathbb{R}^n, \mu)$  for 1.

**Elisabeth Werner (Case Western Reserve University); On the geometry of projective tensor products.** We study a geometric quantity, namely the volume ratio, of the 3-fold projective tensor products  $l_p^n \times l_q^n \times l_r^n$ , with  $1 \leq p \leq q \leq r \leq \infty$ . We obtain asymptotic formulas that are sharp in almost all cases. From the Bourgain-Milman bound on the volume ratio of Banach spaces in terms of their cotype 2 constant, we obtain, as a consequence of our estimates, information on the cotype of these 3-fold projective tensor products. Our results are related to results by Briet, Naor and Regev. They naturally generalize to  $k$ -fold products.

Based on joint work with O. Giladi, J. Prochno, C. Schuett and N. Tomczak-Jaegermann.

**Brett Wick (Washington University at Saint Louis); Commutators, factorization, BMO and the Hardy space.** In this talk we will discuss the connection between functions with bounded mean oscillation (BMO) and commutators of Calderon-Zygmund operators. In particular, we will discuss how to characterize certain BMO spaces related to second order differential operators in terms of Riesz transforms adapted to the operator and how to characterize commutators when acting on weighted Lebesgue spaces. We will also talk about how these results imply corresponding factorization results for the appropriate Hardy space.

**Udeni Wijesooriya (University of Florida); Parametrizing algebraic isopairs.**

An algebraic isopair is a commuting pair of pure isometries that is annihilated by a polynomial defining a distinguished variety. Given a pure algebraic isopair with finite bimultiplicity, there exists a unitary matrix that provides a representation formula for the distinguished variety corresponding to the isopair. In this talk, we discuss a notion of the rank of algebraic isopairs and parametrize the class of pure algebraic isopairs with a fixed rank in terms of the class of above mentioned unitary matrices.

**Abdelrahman Youssef (University of Jordan); commuting Toeplitz operators with bounded symbols.** Various algebraic problems related to Toeplitz operators have been extensively

studied in the literature. One of the most interesting problems in the field is the commuting problem of two Toeplitz operators on the Bergman space of the unit disk. This problem was motivated by the same problem for Toeplitz operators on the Hardy space over the unit circle, which was solved by Brown and Halmos in their seminal paper "Algebraic properties of Toeplitz operators". In this talk, I will present the recent contributions toward solving this problem, then I will show that if a Toeplitz operator on the Bergman space of the unit disk, with right-terminating bounded symbol, commute with another Toeplitz operator whose right-terminating bounded symbol is neither analytic nor coanalytic, then a nontrivial linear combination of both symbols is constant on the unit disk.

**Ziliang Zhang (Vanderbilt University).** In this talk, we consider Toeplitz operators on the Dirichlet space. We characterize the spectrum of Dirichlet Toeplitz operators with symbols in  $\overline{\mathcal{P}} + H_1^\infty$ . We also give a criterion of the asymptotic invertibility of Dirichlet Toeplitz operators with symbols in  $L^{1,\infty} + \overline{H^\infty(\mathbb{D})}$  and establish the Szegő type theorems of Dirichlet Toeplitz determinants.