Dynamics of Mapping Class Groups

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Background

How efficient is this taffy puller?

n-armed taffy pulling action ↔ homeomorphism of an n-punctured plane

Nielsen-Thurston Classification Theorem → to every homeomorphism of a surface we can attach a real number called the stretch factor

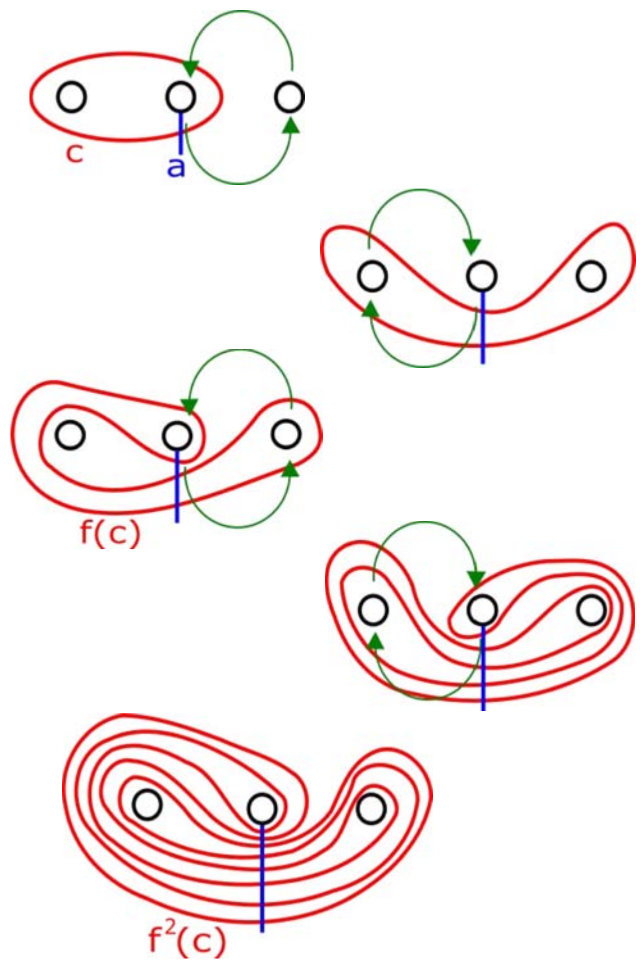
Setup: c = curve,
f = homeomorphism,
a = reference arc.

Stretch Factor = growth rate of $i(f^n(c), a)$

Margalit-Strenner-Yurttas: Quadratic time algorithm that computes the stretch factor.

Our Project: Implement the algorithm.

Example



Stretch Factor = $\phi^2 \approx 2.618$

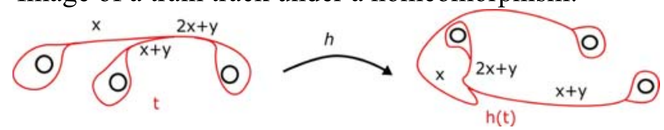
The General Case

Challenge: *How can we compute $f^n(c)$ for arbitrary f, n, and c?*

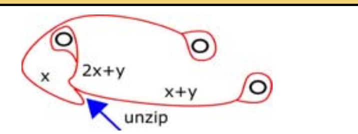
Representing curves as measured train tracks:

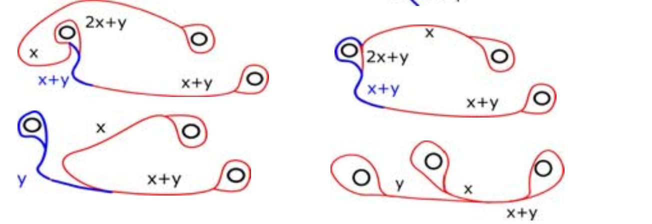
A basis of train tracks:

Image of a train track under a homeomorphism:



Unzip h(t) to obtain a basis train track:





In progress work: Generalize across all surfaces, homeomorphisms, and curves.