

1 HOUR TIME LIMIT  
NO NOTES, NO CALCULATORS  
GOOD LUCK!

(Note that the actual test must be completed in 50 minutes! Thus, even though the format will be similar, you will have less work to do!)

### Problem 1

- (a) (5 points) Find the sum of the series  $\sum_{n=1}^{\infty} (-1)^{n+1} 4^{-n-1}$ .
- (b) (5 points) Compute  $\lim_{x \rightarrow \infty} x[\ln(x+1) - \ln(x-1)]$ .

## Problem 2

(a) (5 points) Does the improper integral  $\int_3^{\infty} \frac{x}{x^{5/2} + 1} dx$  converge? Justify!

(b) (5 points) Is  $\sum_{k=0}^{\infty} (-1)^k \frac{k+1}{k^3+1}$  absolutely convergent? Is it convergent? Justify!

(c) (5 points) If the series at (b) is convergent, find the smallest  $n$  such that the partial sum  $s_n$  approximates the sum of the series within a decimal place.

### Problem 3

- (a) (5 points) Is  $\sum_{k=0}^{\infty} \frac{k+1}{\sqrt{2k^5+3}}$  convergent? Justify!
- (b) (5 points) Prove that  $\sum_{k=0}^{\infty} \frac{k!(3k)!}{(4k)!}$  is convergent.

#### Problem 4

(a) (5 points) Is the improper integral  $\int_{-1}^1 \frac{dx}{x(x+3)}$  convergent? If so, evaluate it.

(b) (5 points) Write down the 4<sup>th</sup> Taylor polynomial associated to the function  $f(x) = -2x^2 + e^{3x}$ .