Problem 1

Consider the matrices

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 2 & 2 & 4 \\ 0 & 2 & -1 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 1 & 2 \end{bmatrix}.$$

(a) (3 points) Write the first column of AB as a linear combination of the columns of A.

(b) (4 points) Find one-to-one parametrizations for $Ker(A)$ and $Img(A)$.

(c) (3 points) Find bases for $Ker(A)$ and $Img(A)$.

Problem 2

Consider the line with

$$\text{base point } \mathbf{x}_0 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \text{ and direction } \mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}.$$

(a) (5 points) Find the parametrization of the plane containing this line and the point

$$\mathbf{p} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix};$$

(b) (3 points) Write down the normal equation of the plane from (a) (the equation of the form $ax + by + cz = d$).

(c) (2 points) Compute the *unit vector* (i.e., of unit length) orthogonal to this plane.

Problem 3

Consider the system of equations

$$\begin{aligned} 2x + ay - z &= 1 \\ x + y - \frac{1}{2}z &= 1 \\ x + 2y + 3z &= 0. \end{aligned}$$

(a) (6 points) For which values of a, if any, does this system have a unique solution? Give the solution for one such value of a.

(b) (2 points) For which values of a, if any, does this system have no solution?

(c) (2 points) For which values of a, if any, does this system have infinitely many solutions?

Problem 4

Consider the matrix

$$A = \begin{bmatrix} -1 & -2 & 1 \\ -2 & 1 & -2 \\ -1 & -2 & 0 \end{bmatrix}.$$

(a) (4 points) Assume A is invertible and compute the third column of A^{-1}.

(b) (6 points) Perform the LU-factorization for A.