

CALCULUS 1501: PRACTICE TEST 3

Problem 1

(a) (4 points) State the Fundamental Theorem of Integral Calculus.

(b) (6 points) Use this theorem to show that

$$\int_0^{\pi} \sin 2x dx = 0.$$

(Hint: Recall that $\sin 2x = 2 \sin x \cos x$. Whose derivative is this?)

Problem 2

(a) (6 points) Use substitution to calculate

$$\int x(x-1)^{2005} dx.$$

(b) (4 points) Use $(\sec x)' = \sec x \tan x$ to compute

$$\int x \sec(x^2) \tan(x^2) dx.$$

Problem 3

(a) (5 points) Suppose that f is continuous on $[0, 1]$ and $\int_0^1 f(x)dx = 0$. Prove that there is at least one number $c \in (0, 1)$ such that $f(c) = 0$.

(b) (5 points) Compute the area of the region enclosed by the parabolae $x + y^2 = 1$ and $y^2 - x = 1$.

Problem 4

(a) (7 points) Consider the trapezoid enclosed by the x -axis, $y = x$, $x = 1$ and $x = 2$. Find the volume of the solid obtained by revolving this region about the y -axis.

(b) (3 points) Use Pappus's Theorem to justify the above calculations.