

CALCULUS 1501: PRACTICE TEST 2

Problem 1

(a) (4 points) State Rolle's Theorem.

(b) (6 points) Use this theorem to show that

$$f(x) = x^5 - \frac{1}{x} + 2005$$

has at most one real zero. Can you then use the Intermediate Value Theorem to find exactly how many real zeroes f has?

Problem 2

(a) (5 points) Is the sequence

$$a_n = \frac{1}{\sqrt{n}} - \frac{1}{n}$$

bounded? If so, what is its greatest lower bound?

(b) (5 points) If

$$b_n = \frac{3^n}{3^n - 1},$$

find

$$\lim_{n \rightarrow \infty} (a_n + b_n).$$

Problem 3

(10 points) A string 30 cm long is to be cut into two pieces, one piece to form a square and the other piece to form a circle. How should the string be cut so as to maximize the sum of the two areas?

Problem 4

(10 points) Sketch the graph of

$$f(x) = \frac{x}{x^2 + 1}$$

by first identifying as many points of interest and features of the graph as possible.