Problem 1

(a) (5 points) Use the fact that

$$\ln(ab) = \ln a + \ln b \text{ for all } a,\ b > 0$$

to prove that

$$e^x e^y = e^{x+y} \text{ for all real numbers } x,\ y.$$

(b) (5 points) Calculate df/dx if

$$f(x) = x^{2x},\ x > 0.$$

Problem 2

(a) (4 points) Use integration by parts to compute

$$\int \ln x dx.$$

(b) (6 points) Use integration by parts twice to calculate

$$\int e^x \sin x dx \text{ and } \int e^x \cos x dx.$$

Problem 3

(a) (5 points) Use decomposition into partial fractions to calculate

$$\int \frac{dx}{x^3 - 2x^2 + x}.$$

(b) (5 points) Use a trigonometric substitution to calculate

$$\int_0^1 \frac{x}{\sqrt{1-x^2}} dx.$$

Problem 4

(a) (7 points) Use integration by parts to compute

$$\int_0^{\pi} \cos^5 x dx.$$

(b) (3 points) Prove that

$$\int_{-\pi/2}^{\pi/2} \sin^{2005} x dx = 0.$$