School of Mathematics
Georgia Tech

Math 1502, SPRING 2008; QUIZZES 1–12

1. Compute
$$\lim_{x\to 0} \frac{e^x + e^{-x} - 2}{1 - \cos 2x} .$$

2. Compute
$$\lim_{x\to 0^+} x^{\sin x} .$$

3. Determine whether the integral
$$\int_e^\infty \frac{\ln x}{x} \mathrm{d}x$$
converges. If so, compute it.

4. Is the series $\sum_{n=1}^{\infty} \frac{\ln\sqrt{n}}{n}$ convergent? Justify!

5. Is the series $\sum_{k=1}^{\infty} \frac{k2^k}{3^k}$ convergent? Justify!

6. Write the Taylor Series expansion of $f(x) = \ln(1 + 2x)$ in powers of $x - 1$.

7. Find the radius and the interval of convergence for the series $\sum_{k=1}^{\infty} \frac{\ln k}{2^k}(x-2)^k$.

8. Consider a *linear transformation* $f : R^3 \to R^2$ such that
$$f(\mathbf{e_1}) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad \mathbf{f}(\mathbf{e_2}) = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \mathbf{f}\left(\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

(a) (5 points) Write down the matrix of the transformation f;
Hint: One needs $f(\mathbf{e_3})$ as well! That can be found by using the equations in the display above and the linearity of f.

(b) (3 points) Compute $f(\mathbf{v})$, where
$$\mathbf{v} = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}.$$

9. Consider the *plane* containing the points $p_0 = \begin{bmatrix} 1 \\ 4 \\ 1 \end{bmatrix}$, $p_1 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$, $p_2 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$. Consider also the *line* passing through $q_0 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $q_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$.

(*a*) (4 points) Write down the *normal* equation of the plane (i.e., in the form $ax + by + cz = d$);

(*b*) (2 points) Write down the *parametric* equations of the line;

(*c*) (2 points) The line intersects the plane at a unique point. Find it!

10. Consider the matrix and the vector

$$A = \begin{bmatrix} 1 & -2 \\ -1 & 1 \\ -2 & -1 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

(*a*) (6 points) Perform the LU-factorization for A;

(*b*) (2 points) Is $A\mathbf{x} = \mathbf{b}$ solvable?

11. Consider the vectors

$$\begin{bmatrix} 1 \\ 1 \\ a \end{bmatrix}, \quad \begin{bmatrix} 1 \\ a \\ 1 \end{bmatrix}, \quad \begin{bmatrix} a \\ 1 \\ 1 \end{bmatrix}.$$

For what values of a do these vectors form a basis for R^3?

12. Consider the subspace $S \subset R^3$ spanned by the vectors

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}, \quad \mathbf{v_2} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

Find the *orthogonal projection* matrices onto S and $S^{\perp}$.

13. Perform the QR-factorization for the matrix

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

and find an orthonormal basis for $Img(A)$.

14. (*a*) (6 points) Compute the determinant of the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 & a \\ 0 & 1 & a & 1 \\ 1 & a & 1 & 0 \\ a & 1 & 0 & 0 \end{bmatrix}.$$

(*b*) (2 points) For which values of a, if any, is A invertible?