

PRACTICE TEST 4
TIME LIMIT: 50 min
CLOSED BOOKS; NO CALCULATORS

GOOD LUCK!

Problem 1

(a) (5 points) Compute the determinant of the matrix

$$A = \begin{bmatrix} 1 & 3 & 5 & 7 & 9 \\ 2 & 4 & 6 & 8 & 10 \\ 1 & 0 & 3 & 2 & 1 \\ 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix}.$$

(b) (5 points) Use the result from (a) to compute the determinant of the matrix

$$B = \begin{bmatrix} 1 & 0 & 3 & 2 & 2 \\ 2 & 4 & 6 & 8 & 12 \\ 0 & 0 & 0 & 1 & 3 \\ 1 & 3 & 5 & 7 & 10 \\ 0 & 0 & 0 & 2 & 2 \end{bmatrix}.$$

Problem 2

Consider the matrix

$$A = \begin{bmatrix} 1 & 3 & 1 \\ 1 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix}.$$

(a) (5 points) Find an orthonormal basis for $\text{Img}(A)$ and perform the QR factorization for A ;

(b) (2 points) What are the dimensions of $\text{Img}(A)$, $\text{Img}(A^t)$, $\text{Ker}(A)$ and $\text{Ker}(A^t)$?

(c) (3 points) Use (a) to write down the *orthogonal projection matrix* onto $\text{Img}(A)$.

Problem 3

Consider the matrix

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 0 & -1 \end{bmatrix} \text{ and the vectors } \mathbf{v} = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} \text{ and } \mathbf{w} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}.$$

- (a) (5 points) Find the orthogonal projection of \mathbf{v} onto $\text{Ker}(A^t)$;
- (b) (5 points) Find the orthogonal projection of \mathbf{w} onto $\text{Ker}(A)$.

Problem 4

Consider the point $P(0, 1, 1)$ and the line l of equations $x = y = 2z$ in \mathbb{R}^3 .

(a) (6 points) Use the *cross product* to find the *normal equation* of the plane containing P and the line l ;

(b) (2 points) Are the points $Q(-2, -2, -1)$ and $R(2, 2, 1)$ on l ? Compute the *area* of the triangle ΔPQR ;

(c) (2 points) Use (b) to find the *distance* between the point P and the line l .