Name______________________________ 9 December 2002

Final Exam

Math 3012 Andrew

Instructions:
1. This is a closed book examination. Calculators may be used.
2. **Be sure to show your work and explain your reasoning and express your answers in a concise and simple form.**
3. Please do any 7 of the 8 problems. Indicate here which problem **you do not want graded** __________.

1. (25) A *hand* in the game of *bridge* consists of 13 cards selected without regard to order from a standard deck of 52 cards. These 52 cards are classified into four thirteen card *suits*: spades, hearts, diamonds, and clubs.

   a. How many bridge hands contain exactly 5 hearts?

   b. How many bridge hands contain the Ace of hearts, the Queen of hearts, and exactly 3 other hearts?

   c. How many bridge hands contain no more than 4 hearts?

2. (25) Show that if 5 points are selected in the interior of a square of area one, there are at least two whose distance from each other is strictly less than $1/\sqrt{2}$.

3. (25) Professor Andrew is trying to figure out how to make grading assignments for his four teaching assistants. Each of the four TAs will grade exactly one of the four test problems. Unfortunately, TA1 refuses to grade problem 1, TA2 refuses to grade either problem 1 or problem 2, TA3 will not grade problem 4, and TA4 will not grade either problem 2 or 3.

   a. Draw a chess board that will help determine how many ways there are to make grading assignments under these difficult conditions.

b. Calculate the rook polynomial of the chessboard in part a.

c. Use the results of parts a and b to determine how many ways there are to make grading assignments under these conditions.

4. Calculate the solution of the recurrence relation

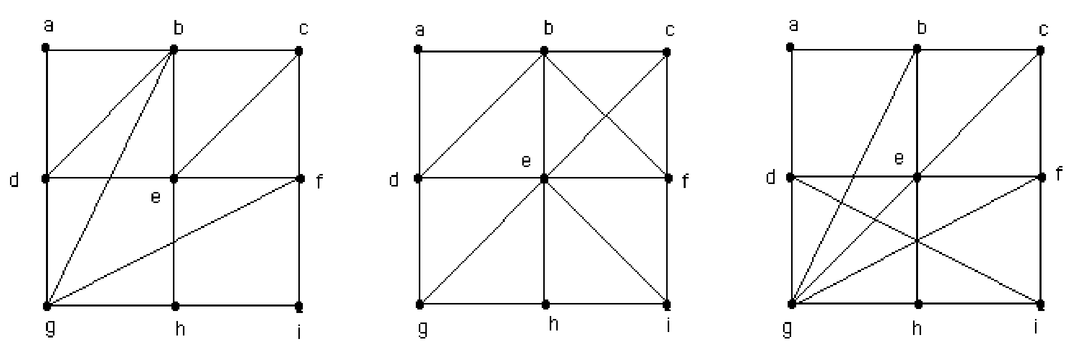
$$a_n - a_{n-1} - 2\,a_{n-2} = 3\left(2^n\right),\quad n \geq 2$$
$$a_0 = 0$$
$$a_1 = 0$$

5. (25) Use generating functions to determine the number of ways 25 college freshman can be assigned to 4 intramural sports teams if

a. At least four must be assigned to each team.

b. At least four and no more than nine may be assigned to each team.

6. (25) Let *G, H, I* be the graphs sketched below.



G *H* *I*

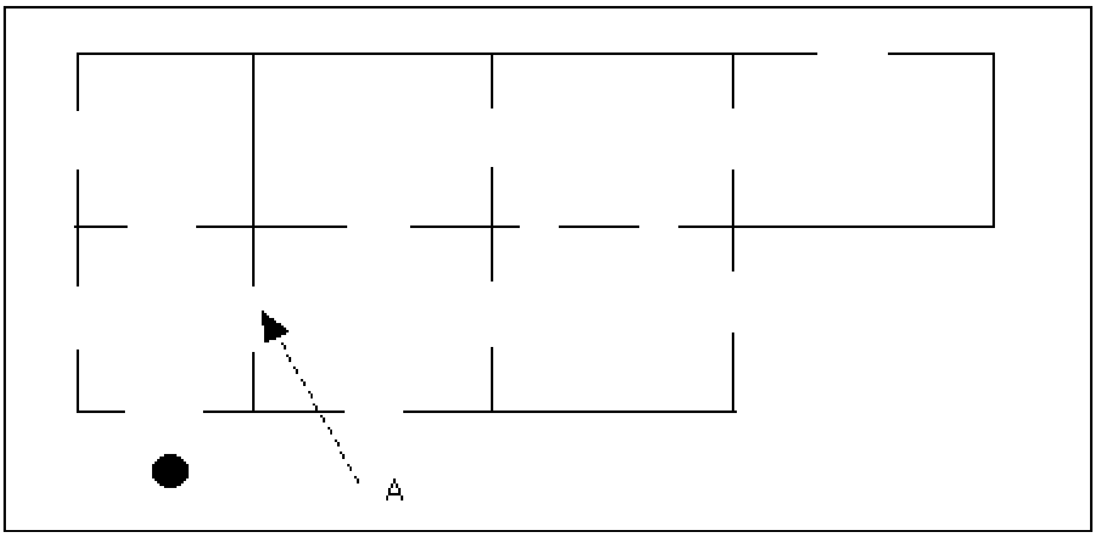
a. Calculate the chromatic number of *G*.

b. Calculate the chromatic number of *H*.

c. Determine whether any pairs of these graphs are isomorphic. Explain.

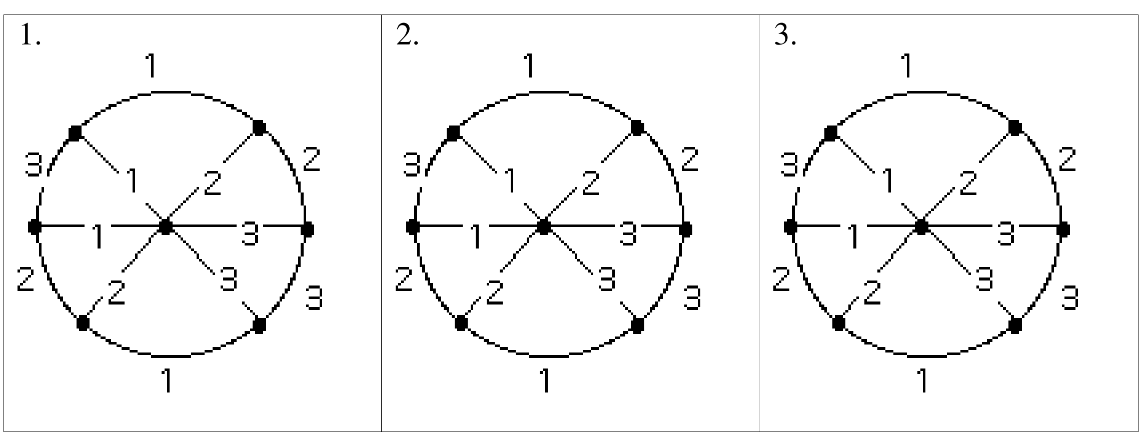
7. (25) (Problem 11.3.21) When visiting a chamber of horrors, Paul and David try to figure out whether they can travel through the seven rooms and surrounding corridor of the attraction without passing through any door more than once.

a. If they must start from the indicated position in the corridor, can they accomplish their goal?



b. Discuss as completely as you can the situation in which the door labeled *A* is blocked by a terrible monster and they cannot pass through it.

8. (25) a. Using either the Kruskal or the Pim algorithm, find a minimum weight spanning tree for the graph sketched below. Illustrate a single step of the algorithm on each picture, and state which algorithm you are using.



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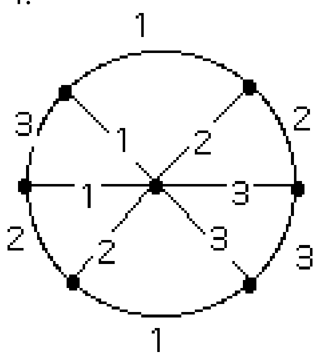
4.



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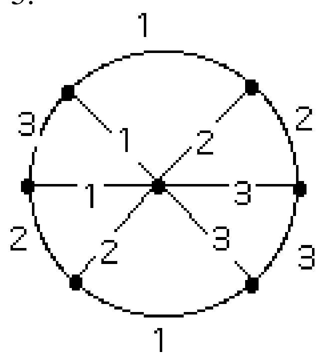
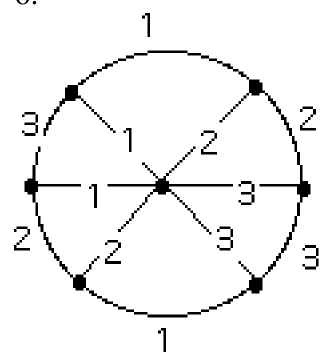
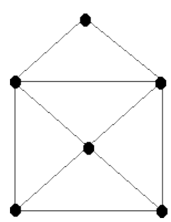


6.

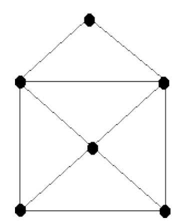


b. Assign weights 1, 1, 1, 2, 2, 2, 3, 3, 4, 4 to the graph below so that it has



A unique minimal weight spanning tree.

Indicate the edges of the minimal weight spanning tree and calculate its weight.



More than one minimal weight spanning tree.

Indicate the edges of **two** minimal weight spanning trees and calculate their weights.

Answers.

1. a. $\binom{13}{5}\binom{39}{8}$ b. $\binom{11}{3}\binom{39}{8}$ c. $\sum_{i=0}^{4}\binom{13}{i}\binom{39}{13-i}$

2. Subdivide the square and use the pigeon-hole principle.

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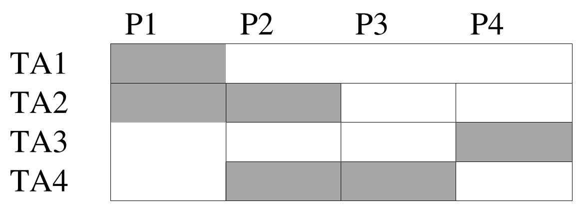
3. a.



b. $1 + 6x + 11x^2 + 7x^3 + x^4$ c. 4

4. $a_n = -\frac{4}{3}(2)^n + \frac{4}{3}(-1)^n + n2^{n+1}$

5. a. 220 b. 140

6. a. 3 b. 4 c. No pairs are isomorphic

7. Make a graph with a vertex for each room and one for the corridor and add an edge for each door.