Instructions: 1. This is a closed book examination. Calculators may be used.
2. Please do all 5 problems. They count equally.
3. **Be sure to show your work and explain your reasoning.**

1. a. *Forming a Committee*. In how many ways can an 8 person committee be selected from a combinatorics class with 38 students?

   b. *Forming a Committee with officers.* In how many ways can an 8 person committee be selected from a 38 student class if one committee member is to be designated *chairperson* and a different committee member is to be designated *secretary*?

2. a. In how many ways can the 38 person class be divided into 5 committees, if committee A has 5 members, committee B has 6 members, committee C has 9 members, committee D has 7 members, and committee E has 11 members?

   b. In how many ways can 5 children share 45 identical red marbles and 20 identical blue marbles, if each child must have at least 4 red marbles?

3. Use mathematical induction to prove that for each nonnegative integer n,

$$1 + x + x^2 + \cdots x^n = \frac{1 - x^{n+1}}{1 - x} . \quad (x \neq 1)$$

4. a. Use the Euclidean Algorithm to compute $d = gcd(450,264)$ and to express

$$d = 450\,x + 264\,y .$$

   b. Does the equation $450\,x + 264\,y = 99$ have any positive integer solutions x, y? Explain.

5. Before the game starts, a fan at a college basketball game takes a quick look at the players for the home team, and announces that there are two players who are the same age and the same height (measured to the nearest inch). Assuming that no player is younger than 18, no player is older than 21, and the heights range from 6 feet 10 inches to 7 feet 3 inches, what is the smallest number of players the fan could have seen?

Answers.

1. a. $\binom{38}{8}$ b. $(8)(7)\binom{38}{8}$

2. a. $\binom{38}{5\ 6\ 9\ 7\ 11} = \frac{38!}{5!\,6!\,9!\,7!\,11!}$

   b. The blue marbles may be shared in $\binom{24}{4}$ ways. First distribute 4 red marbles to each child and there are then $\binom{29}{4}$ ways to distribute the remaining red marbles. Finally, there are $\binom{24}{4}\binom{29}{4}$ to distribute the red and blue marbles, with each child receiving at least 4 red marbles.

4. a. gcd = 6 b. No, since 6 does not divide 99

5. Since there are 4 possible ages (18, 19, 20, 21) and 6 possible heights, there are 24 distinct height/age ordered pairs. He must have seen at least 25 players.