Name________________________________ 25 October 2002

Hour Test 2

Math 3012 Andrew

Instructions:

1. This is a closed book examination. Calculators may be used.
2. Please do all 4 problems. They count equally.
3. **Be sure to show your work and explain your reasoning.**

1. Use the method of inclusion/exclusion to determine the number of ways Professor Andrew may assign points to the **five** problems on a test if

   a. The value of each problem must be a multiple of **5**,
   b. The sum of the problem values must be **120**,
   c. No problem may be worth less than **5** points, and
   d. No problem may be worth more than **30** points.

2. Three people eating at a restaurant decide to have different dishes, but each also has certain ordering preferences. Muriel will not order Neeps, nor will she order Tatties. Fred will not order Tatties or Steak au Poivre, and Marion will not order Haggis or Neeps.

   a. Indicate the chess board that would be used to determine in how many ways these three difficult patrons could select a dinner menu from the four choices of foods listed.

	N	H	SoP	T
Mu				
F				
Ma				

   b. Calculate the rook polynomial of the chessboard you found in Part a.

   c. In what country do men wear kilts and eat Haggis, Neeps, and Tatties?

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3. (Problem 12, page 399) Two cases of soft drinks, 24 bottles of one type and 24 of another, are distributed among five surveyors who are conducting taste tests. **Use generating functions** to determine in how many ways the 48 bottles can be distributed so that each surveyor gets at least two bottles of each type. (Please express your answer in terms of binomial coefficients $\binom{a}{b}$ with $a > 0$.)

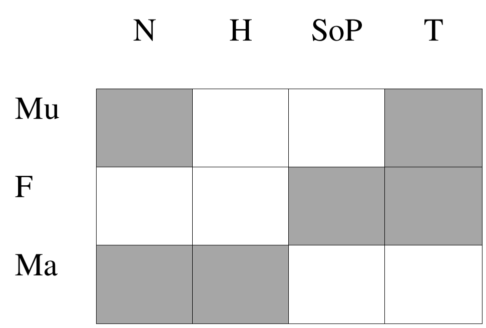
4. Solve the recurrence relation

$$2a_n + 3a_{n-1} + a_{n-2} = 36n$$
$$a_0 = 10$$
$$a_1 = 8$$

Answers.

1. 205

2. a.



b. $p(x) = 1 + 6x + 10x^2 + 4x^3$ c. Scotland, but all answers received full credit.

3. To count the number of ways to distribute the first case of soda, we seek the coefficient of x^{24} in $(x^2 + x^3 + \cdots)^5 = x^{10}(1-x)^{-5}$, which is $\binom{18}{14}$. This is also the number of ways to distribute the second case, so the answer to this problem is $\binom{18}{14}^2$.

4. $a_n = 4\left(-\frac{1}{2}\right)^n + (-1)^n + 6n + 5$