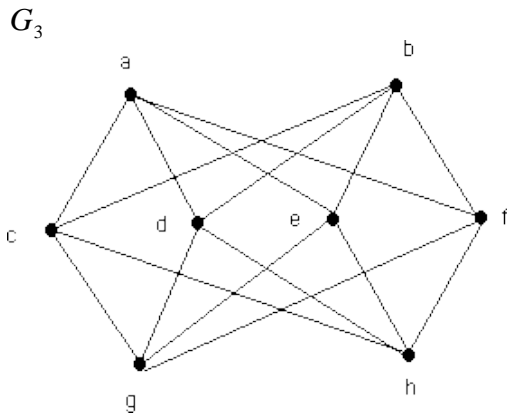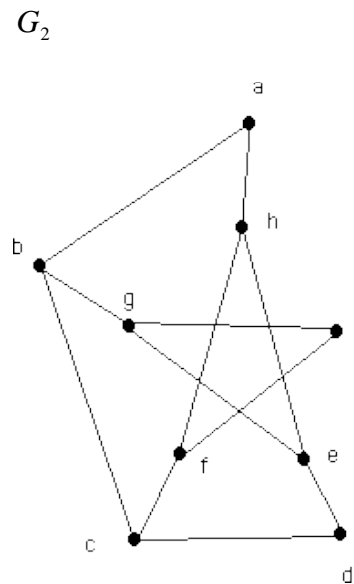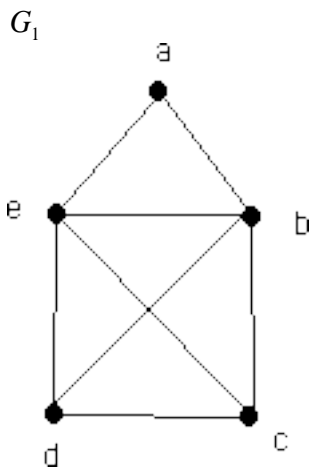Instructions: 1. This is a closed book examination. Calculators may be used.
                2. Please do all 5 problems. Points are as indicated.
                3. **Be sure to show your work and explain your reasoning.**
                4. Several of the problems refer to graphs G_1, G_2, and G_3 shown on this
                    page. You may wish to use the extra copies of the graphs printed at
                    the end of the test, but if you do, please make it easy for me to find all
                    work you wish to have graded.

G_1

G_2

G_3

1. (5) The location of the final exam in this course is _____.

   (If you think it's in th Physics Building, be *very* sure to come to class the day this test is discussed!)

2. (25) Determine which of graphs G_2 and G_3

   (a) have a Hamilton cycle

   (b) have an Euler circuit.

   Be sure to explain your reasoning.

3. (25) Which of graphs $K_{2,5}$, K_6, G_1 and G_2, are planar? Either exhibit a planar embedding or a subgraph homeomorphic to K_5 or K_{33}.

4. (25) Find

   a. The chromatic polynomial of G_1.

   b. The chromatic number of G_1.

   c. The number of proper colorings of G_1 using 4 colors.

5. (25) Find all nonisomorphic spanning trees of the complete bipartite graph K_{23}. Be sure to convince me that you have indeed found them all.

Answers.

2. G_2 has a Hamilton cycle (dcbahfiged) but does not have an Euler circuit since deg(b) is odd. G_3 has a Hamilton cycle by Ore's Theorem and also has an Euler circuit.

3. G_1 is planar, G_2 contains a homeomorph of $K_{3,3}$ and hence is not planar.

4. a. $(\lambda)(\lambda - 1)(\lambda - 2)^2(\lambda - 3)$　　　　　b. 4　　c. 48

5. There are two. One has maximum vertex degree 3 and one has maximum vertex degree 2.