

## Instructions:

1. You may use the assigned text by Hogg and Tanis, except that you may not use the answers section. Calculators may be used.
  2. If you use a table in the text to, state which table you are using and the page on which it appears. If you use a built in function on your calculator instead of referring to a table in the text, state which function, and the calculator model number.
  3. Please do all problems. Problems count equally.
  4. **Be sure to show your work and explain your reasoning.**
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1. (25) a. A gambler rolls two dice and counts the sums of the spots. What's the probability that the sum of the spots is 5, given that it is either 5 or 7?  
  
b. Orders for pastrami sandwiches on rye bread are placed at a delicatessen according to a Poisson process with mean 4 orders per hour. What's the probability that during a 3 hour period at least 10, but no more than 20 orders for these sandwiches are placed?  
  
c. Cards are drawn from a standard deck. Compute the probability that the sixth time a spade is drawn occurs on the 13th draw if (i) the cards are drawn with replacement and (ii) without replacement.
  2. (25) A large bin at a popular garden center contains numerous bags of tulip bulbs. 75% of the bags are "Dutch Uncle" brand and contain five red and ten yellow bulbs, while the remaining bags are "Dutch Aunt" brand and contain fifteen red and ten yellow. A bag is selected at random, and one bulb is chosen from the bag. Find
    - a. The probability the bulb selected is red.
    - b. The probability the bulb selected is yellow.
    - c. The probability that the bag selected is "Dutch Aunt" brand, given that the bulb is red.

3. (25) Suppose  $X$  has exponential distribution with mean  $\theta = 30$ .
- Compute  $P(10 < X < 20)$
  - Compute  $P(30 < X)$
  - Compute  $P(X > 50 \mid X > 20)$
  - If  $X_1, X_2, \dots, X_{10}$  are a random sample from this distribution, approximate  $P(X_1 + \dots + X_{10} > 350)$ .
4. (25) Random variables  $X$  and  $Y$  have joint probability density function  $f(x, y) = \frac{5y}{2}$  for  $\{(x, y) \mid 0 \leq x \leq 1 \text{ and } x^2 \leq y \leq 1\}$ .
- Compute the marginal probability density functions  $f_1(x)$  and  $f_2(y)$ .
  - Compute the means  $\mu_X$  and  $\mu_Y$ .
  - Compute  $P(X < Y)$ .
5. (25) Suppose a random sample of  $n$  from a normal distribution with unknown mean and unknown variance produces a sample mean  $\bar{X} = 75.31$  and sample variance  $S_X^2 = 210$ .
- Find the endpoints and length of a 90% confidence interval for the unknown variance  $\sigma^2$ , assuming  $n = 12$ . Please express your answers in decimals.
  - Find the endpoints and length of a 90% confidence interval for the unknown variance  $\sigma^2$ , assuming  $n = 30$ . Please express your answers in decimals.
6. (25) An advertiser for "Crust Gumpaste" (first advertised in the December 1958 *Mad Magazine*) claims that 75% of dentists recommends "Crust" for their patients having no teeth. A popular Atlanta consumer advocate doubts this claim, and tests the hypothesis  $H_0 : p = .75$  against the alternative  $H_0 : p < .75$ . He finds that 261 dentists in a sample of 370 recommend "Crust" for their toothless patients. Which hypothesis is accepted for significance level
- $\alpha = .05$
  - $\alpha = .01$

Answers.

1. a.  $\frac{4}{10}$       b. .746      c. i.  $\binom{12}{5} \left(\frac{1}{4}\right)^6 \left(\frac{3}{4}\right)^7$     ii.  $\left( \frac{\binom{13}{5} \binom{39}{7}}{\binom{52}{12}} \right) \left( \frac{8}{40} \right)$

2. a.  $\frac{2}{5}$       b.  $\frac{3}{5}$       c.  $\frac{3}{8}$

3. a.  $e^{-1/3} - e^{-2/3}$       b.  $e^{-1}$       c.  $e^{-1}$       d. .2991

4. a.  $f_1(x) = \frac{5}{4}(1-x^4)$      $f_2(y) = \frac{5}{2}y^{3/2}$       b.  $\mu_x = \frac{5}{12}$ ,  $\mu_y = \frac{5}{7}$

c.  $\frac{5}{6}$

5. a. [117.4, 504.9], 387.5      b. [143.1, 343.9], 200.7

6. We reject  $H_0$  at the .05 level. We do not reject  $H_0$  at the .01 level.