

- Instructions:
1. This is a closed book examination. Calculators may be used.
 2. Please do all problems. Problems count equally.
 3. **Be sure to show your work and explain your reasoning.**
 4. Recall that a deck of playing cards contains 52 cards, each marked with one of 4 suits. The suits are named *spades*, *hearts*, *diamonds*, *clubs*, and there are 13 cards in each suit. A die has six faces, labeled 1 through 6, which are equally likely when the die is tossed.
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1. A probability class has 6 students who are on the football team and 30 students who are not. Exactly 5 students will receive the grade of "A," and these students are to be chosen at random.
 - a. What is the probability that all A's are given to members of the football team?
 - b. What is the probability that no A's are given to members of the football team?
 - c. What is the probability that exactly two A's are given to members of the football team?
 - d. What is the probability that at most two A's are given to members of the football team?
 - e. Explain your reasoning in a few short sentences.
 2. Ten cards are drawn, with replacement, from a standard deck.
 - a. What is the probability that the second and fourth cards are hearts?
 - b. What is the probability that the second and fourth cards, and no others, are hearts?
 - c. What is the probability that exactly two of the cards are hearts?
 - d. What is the probability that the third heart is the 10th card drawn?
 - e. Explain your reasoning in a few short sentences.

3. Urn 1 contains 5 red, 4 blue, 3 green, and 2 white balls.
Urn 2 contains 5 red, 2 blue, 2 green, and 5 white balls.
A ball is chosen from Urn i , and the random variable X_i is given the value 1 if the ball is red, 2 if the ball is blue, 3 if the ball is green, and 4 if the ball is white.
- Compute the probability mass functions for X_1 and X_2 , and sketch a bar graph for each.
 - Compute the mean and variance of X_1 and X_2 .
 - Explain the relationship between the bar graphs and variances.
4. a. Just as in problem 3,
Urn 1 contains 5 red, 4 blue, 3 green, and 2 white balls, and
Urn 2 contains 5 red, 2 blue, 2 green, and 5 white balls.
A single die is rolled, and if a one, two, three, four, or five appears a ball is selected from Urn 1, otherwise a ball is selected from Urn 2. Assuming the ball chosen is white, what is the probability that it came from Urn 2?
- In the game of "craps," a man rolls two dice simultaneously. If the sum of the spots on the first roll is 7 or 11, he wins immediately. If it is 2, 3, or 12, he loses immediately. In all other cases, the sum of the spots on the first roll is called his "point," and he continues rolling until he rolls either his point or a 7. He loses if he rolls a 7 first, and wins if he rolls his point first. Compute the probability that he wins, given that his point was 8. Hint: First, explain why this is the probability that a roll is an 8, given that it is either a 7 or an 8.

Answers. I list only answers. You, of course, must explain your work.

$$1. \quad a. \frac{\binom{6}{5}\binom{30}{0}}{\binom{36}{5}} \quad b. \frac{\binom{6}{0}\binom{30}{5}}{\binom{36}{5}} \quad c. \frac{\binom{6}{2}\binom{30}{3}}{\binom{36}{5}} \quad d. \frac{\binom{6}{0}\binom{30}{5} + \binom{6}{1}\binom{30}{4} + \binom{6}{2}\binom{30}{3}}{\binom{36}{5}}$$

d. Don't forget your explanation.

$$2. \quad a. \left(\frac{1}{4}\right)^2 \quad b. \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^8 \quad c. \binom{10}{2} \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^8 \quad d. \binom{9}{2} \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^7 \left(\frac{1}{4}\right)$$

$$3. \quad b. \mu_1 = \frac{30}{14}, \mu_2 = \frac{5}{2}, \sigma_1^2 = \frac{55}{49}, \sigma_2^2 = \frac{47}{28}$$

c. The variance of X_2 is larger, since the bar graphs show X_1 is more concentrated near its mean.

$$4. \quad a. \frac{1}{3} \quad b. \frac{5}{11}$$