

## Instructions:

1. You may use the assigned text by Hogg and Tanis, except that you may not use the answers section. Calculators may be used.
  2. If you use a table in the text to compute a probability, state which table you are using and the page on which it appears. If you compute a probability using a built in function on your calculator, state which function, and the calculator model number.
  3. Please do all problems. Problems count equally.
  4. **Be sure to show your work and explain your reasoning.**
1. When she sits on the front porch, Mrs. Andrew is bitten by mosquitoes according to a Poisson process, with an average of two bites per minute. Please express your answers in decimals.
    - a. What is the probability that she is bitten exactly 8 times during her first five minutes on the porch?
    - b. What is the probability that she is bitten exactly 8 times during the first five minutes and exactly 3 times during the second five minutes?
    - c. What is the probability that the 5th mosquito bite occurs after she has been on the porch at least 5 minutes?
  2. The random variable  $X$  has cumulative distribution function
 
$$F(x) = \begin{cases} 0 & x \leq 0 \\ 4x^3 - 3x^4 & 0 \leq x \leq 1 \\ 1 & 1 \leq x \end{cases}$$
    - a. Find the probability density function of  $X$ .
    - b. Calculate the mean and variance of  $X$ .
    - c. Assuming  $X_1, X_2, \dots, X_n$  are independent random variables with cumulative distribution function

$$F(x) = \begin{cases} 0 & x \leq 0 \\ 4x^3 - 3x^4 & 0 \leq x \leq 1, \\ 1 & 1 \leq x \end{cases}$$

calculate the mean and variance of

i.  $\frac{X_1 + X_2 + \cdots + X_n}{n}$

ii.  $X_1 + 2X_2 + 3X_3$

3. Suppose  $X$  has normal distribution with mean 30 and standard deviation 10.

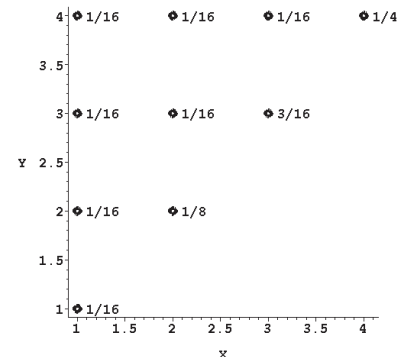
a. i. Find  $P(10 < X < 40)$  (Please express your answer in decimal form)

ii. Find  $x$  such that  $P(x < X) = .0025$  (Please express your answer in decimal form)

b. If  $X_1, X_2, \dots, X_n$  are independent  $N(30, 10)$  random variables, find the probability that  $X_{10}$  is the fourth of the  $X_i$  that exceeds 40.

4. Two 4-sided dice, one red and one blue, are rolled. The random variable  $X$  is the number of spots showing on the blue die, and  $Y$  is the maximum of the number of spots showing on the red die and the number of spots showing on the blue die. The space and the joint probability mass function of  $(X, Y)$  are shown in the figure below.

a. Explain why  $f(2, 2) = \frac{1}{8}$ .



b. Indicate the marginal distribution of  $X$  and the marginal distribution of  $Y$  in the margins of the figure above.

c. Find the probability that the maximum number of spots showing on the two dice is the number of spots showing on the blue die.

d. Calculate the mean of  $X$ , the mean of  $Y$ , and the expected value of  $X$  given that  $Y = 4$ .

Answers.

1. a. .113    b. .000791    c. .029

2. a.  $f(x) = \begin{cases} 12x^2 - 12x^3 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$     b.  $\mu = 3/5$ ,  $\sigma^2 = 1/25$

c.i.  $\mu = 3/5$ ,  $\sigma^2 = 1/25n$     ii.  $\mu = 18/5$ ,  $\sigma^2 = 14/25$

3. a. i. .8185    ii. 58.1

b.  $\binom{9}{3} (.1587)^4 (.8413)^6 = .01889$

4. b. Marginal distribution of X is constant  $1/4$ . Marginal distribution of Y is

$$P(Y=y) = \frac{2y-1}{16} \text{ for } y = 1, 2, 3, 4.$$

c.  $5/8$

d.  $\mu_x = 5/2$ ,  $\mu_y = 25/8$ ,  $E(X|Y=4) = 22/7$