Instructions:

1. You may use the assigned text by Hogg and Tanis, except that you may not use the answers section. Calculators may be used.
2. If you use a table in the text to, state which table you are using and the page on which it appears. If you use a built in function on your calculator instead of referring to a table in the text, state which function, and the calculator model number.
3. Please do all problems. Problems count equally.
4. **Be sure to show your work and explain your reasoning.**

1. (25) Suppose that $X_1, X_2, \ldots$ are a random sample from a continuous type distribution with mean 8 and variance 4.

    a. Compute the mean and variance of $X_1 - 2X_2 + 2X_3 + X_4$.

    b. With $Y = X_1 + X_2 + \mathsf{L} + X_{100}$, estimate $P(Y > 830)$.

2. (25) Consider the probability distribution function

$$f(x;\theta) = \frac{1}{\theta} x^{\left(\frac{1-\theta}{\theta}\right)},$$

$$0 < x < 1, \, 0 < \theta < \infty.$$

and show that the maximum likelihood estimator for θ is $\hat{\theta} = -\frac{1}{n}\sum_{i=1}^{n} \log(x_i)$ Be sure to convince me you've found a *maximum*.

3. (25) Let $X_1 = 6$, $X_2 = 1$, $X_3 = 8$, $X_4 = 3$, $X_5 = 7$.

    a. Find a 90% confidence interval for μ, assuming these X_i are random sample from a normal distribution with unknown mean μ and variance 9.

    b. Find a 90% confidence interval for μ, assuming these X_i are random sample from a normal distribution with unknown mean μ and unknown variance.

4. (25) Suppose that the IQ of university professors in normally distributed with variance 64.

   a. Find the sample size needed to be 90% sure that the maximum error in using the sample average to estimate the unknown mean is 4.

   b. What sample size needed to be 90% sure that the maximum error in using the sample average to estimate the unknown mean is 2?

Answers.

1. a. $\mu = 16$, $\sigma^2 = 40$ b. .0668

3. a. [2.79, 7.21]   b. [2.22, 7,78]

4. a. 11   b. 44