Name________________________________ 29 November 2000

Math 4305 Andrew Hour Test 3

Instructions:

1. Please do all four problems.
2. Closed book, calculators may be used, except on problem 4.
3. Be sure to explain your work.

1. (25) Determine whether the function

$$f(x,y,z) = 2x^2 + 2xy + 3y^2 - 4yz + 2z^2 + 4$$

has a maximum, minimum, or saddle point at (0,0,0).

2. (25) The matrix $\mathbf{B} = \begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 2 & 1 & 1 \\ 1 & 1 & 2 & 0 \\ 1 & 1 & 0 & 3 \end{pmatrix}$, being symmetric, has real eigenvalues.

   a. Use the pivots of B to determine how many negative eigenvalues B has.

   b. Use the Gershgorin Circles Theorem to find real numbers α and β such that every eigenvalue λ of **B** satisfies $\alpha \le \lambda \le \beta$.

3. (25) a. With the Euclidean norm $\|\mathbf{x}\|_2 = \left(\sum x_i^2\right)^{1/2}$ used for vectors, let the norm of a matrix **A** be defined by $\|\mathbf{A}\| = \max\left\{ \frac{\|\mathbf{Ax}\|_2}{\|\mathbf{x}\|_2} : \mathbf{x} \ne \mathbf{0} \right\}$.

   a. Show that if λ is an eigenvalue of **A** then $|\lambda| \le \|\mathbf{A}\|$.

   b. Give an example of a matrix **A** for which $|\lambda| < \|\mathbf{A}\|$ for *every* eigenvalue λ of **A**.

   c. Does there exist a symmetric matrix with $|\lambda| < \|\mathbf{A}\|$ for *every* eigenvalue λ of **A**?

4. (25) Starting with $\mathbf{x}_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, compute the first two Jacobi iterates $\mathbf{x}_1$ and $\mathbf{x}_2$ for the equation **A x** = **b**, with $\mathbf{A} = \begin{pmatrix} 5 & 1 \\ 1 & 2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 10 \\ 6 \end{pmatrix}$. How do you know the Jacobi iterates will converge to the solution of **A x** = **b**?

Answers.

1. minimum

2. a. **B** has one negative eigenvalue. b. $-1 \le \lambda \le 5$

3. a. If $Ax = \lambda x$, then $\|Ax\| = \|\lambda x\| = |\lambda|\|x\|$ so if $\|x\| = 1$,

$$|\lambda| = \|Ax\| \le \max\{\|Ay\| : \|y\| = 1\} = \|A\|$$

b. $\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ c. No

4. $x_1 = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $x_2 = \begin{pmatrix} 7/5 \\ 2 \end{pmatrix}$