

Theorem SP-DO-C

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```
In[1]:= << goedel54.11a; << tools.m

:Package Title: goedel54.11a      2004 February 11 at 5:30 p.m.

It is now: 2004 Feb 13 at 15:48

Loading Simplification Rules

TOOLS.M                          Revised 2004 January 3

weightlimit = 40
```

summary

This notebook contains a two-step derivation of Theorem **SP-DO-C**, which was proved using **Otter** 1997 December 15 using essentially the same technique. The theorem says that if x is a set, then the domain of the complement of x is the universal class V .

derivation

This first step can be carried out either by a blind application of **AssertTest**, or as follows:

```
In[2]:= SubstTest[equal, 0, dif[y, x], y -> cart[complement[domain[complement[x]]], V] // Reverse
Out[2]= subclass[cart[complement[domain[complement[x]]], V], x] == True
```

This made into a temporary rewrite rule.

```
In[3]:= (% /. x -> x_) /. Equal -> SetDelayed
```

The second step uses the subset theorem: a subclass of a set is a set.

```
In[4]:= Map[implies[member[x, y], #] &,
  SubstTest[implies, and[subclass[u, v], member[v, V]], member[u, V],
  {u -> cart[complement[domain[complement[x]]], V], v -> x}]
Out[4]= or[equal[V, domain[complement[x]]], not[member[x, y]]] == True
```

Because the following rewrite rule will be made permanent, an extra variable y has been introduced to facilitate pattern matching for applications of this rule.

```
In[5]:= or[equal[V, domain[complement[x_]], not[member[x_, y_]]] := True
```