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**Johan G. F. Belinfante** ([belinfan@math.gatech.edu](mailto:belinfan@math.gatech.edu)), School of Mathematics, Georgia Institute of Technology, Atlanta, GA 30332-0160. *On the Automated Hiding of Quantifiers in Gödel's Class Theory.*

Kurt Gödel's algorithm for class-formation generally produces complex constructions for simple classes. A modification of his algorithm, bypassing his use of Kuratowski's definition of ordered pairs, and including many simplification rules, was implemented in **Mathematica** as a practical tool for formulating theorems of class theory to use with William McCune's automated theorem prover **Otter**, which uses first order logic. The program has been set up to automatically hide all quantifiers over set variables in assertions. For example, if one inputs the assertion that  $z$  is a relation whose vertical sections are pairwise disjoint or equal,

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assert[forall[x,y,or[equal[image[z, singleton[x]], image[z, singleton[y]]],
                    disjoint[image[z, singleton[x]], image[z, singleton[y]]]]]]]
```

the output is the equivalent quantifier-free assertion  $z \circ \text{inverse}[z] \circ z \subset z$ . Similarly, if one inputs the statement that for any two vertical sections of  $z$ , one of the vertical sections is contained in the other, the output is the statement  $z \circ \text{complement}[\text{inverse}[z]] \circ z \subset z$ . Exploiting such techniques, the presenter has used **Otter** to prove many theorems in ordinal number theory, starting from John Isbell's elegant definition of an ordinal number.

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