No books or notes allowed. No laptop, graphic calculator or wireless devices allowed. Write clearly.

Name: ______________________________

1. (10 points) Given $f(x) = x(1-x)$ let A be the finite region of the plane contained between $-f(x)$ and $f(x)$. That is A is defined by;

$$0 \le x \le 1 \qquad\qquad -x(1-x) \le y \le x(1-x)$$

Find the area of A and the volume of the solid S obtained by rotating A around the y axis (use the symmetries of A to simplify the computations).

Solution:

The area of A is given by

$$\text{Area}(A) = 2\int_0^1 x(1-x)dx = 2\left(\frac{x^2}{2}\bigg|_0^1 - \frac{x^3}{3}\bigg|_0^1\right) = \frac{1}{3}.$$

Observe that A is symmetric with respect to the x axis and the vertical line $x = 1/2$ so that the centroid of A is $\bar{x} - 1/2$ and $\bar{y} = 0$. Form Pappus theorem it follows that

$$\text{Vol}(S) = 2\pi\bar{x}\text{Area}(A) = \frac{\pi}{3}.$$

Without using Pappus Theorem, the volume can be computed with the shells method and is given by

$$\text{Vol}(S) = 4\pi\int_0^1 x^2(1-x)dx = 4\pi\left(\frac{x^3}{3}\bigg|_0^1 - \frac{x^4}{4}\bigg|_0^1\right) = \frac{\pi}{3}.$$