Name_____ 16 September 2004

Teaching Assistant_____ Hour Test 1
Math 1501 C3 - C6 Andrew

Instructions: 1. Closed book.
2. Show your work and explain your answers and reasoning.
3. Calculators may be used, but pay particular attention to instruction 2. **To receive credit, you must show your work.** Unexplained answers, and answers not supported by the work you show, will not receive credit.
4. Express your answers in simplified form.

1. (25) a. Let $f(x) = \begin{cases} x^2 \sin(1/x) & x \neq 0 \\ 0 & x = 0 \end{cases}$. Use the definition of derivative to compute $f'(0)$.

b. Evaluate

i. $\lim_{x \to 3} \dfrac{x^2 + x - 12}{x^2 - x - 6}$

ii. $\lim_{x \to 0} \dfrac{\sin(3x)}{2x}$

2. (25) Calculate

a. $f'(x)$ for $f(x) = \dfrac{x}{x^4 + 9}$

b. $\dfrac{dg}{dx}$ for $g(x) = \sin^3\left(x^2 + 1\right)$

c. $\dfrac{dx}{dt}$ for $x(t) = t^2\sqrt{4 - t^2}$

d. $\dfrac{dy}{dx}$ at the point (2, - 5) if $4x^2 + 2xy + 5y^2 = 121$.

3. (25) a. Find the equation of the line L that is tangent to the graph of $f(x) = \frac{x^2}{4}$ at the point P for which $x = 4$.

   b. Sketch the graph of $f(x) = \frac{x^2}{4}$ and the line L on the axes below. Label the point Q, where the tangent line L intersects the y-axis and the point F (0,1).

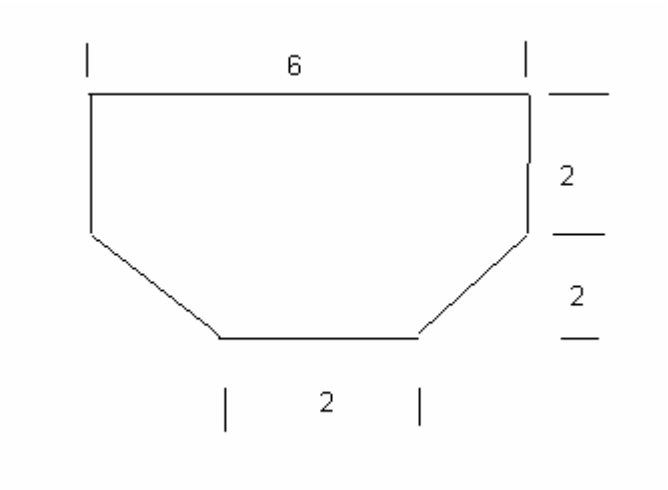
   c. Show that the distance from the point F (0,1) to P equals the distance from F to Q.

   d. Does the conclusion of part *c* remain true if P is replaced by the point where $x = a$, $a > 0$ and Q is the y-intercept of the tangent line at the relocated P?

4. (25) A trough has cross-section as shown below, and is 10 feet long. It is being filled with water at the rate of 5 cubic feet per minute. At what rate is the depth of water changing when

   a. The depth of water in the trough is 1 foot?

   b. The depth of water in the trough is 3 feet?



Answers.

1. a. $f'(0) = 0$. b. $7/5$ c. $3/2$

2. a. $f'(x) = \dfrac{-3x^3 + 9}{\left(x^4 + 9\right)^2}$ b. $\dfrac{dg}{dx} = 6x\sin^2(x^2+1)\cos(x^2+1)$

c. $\dfrac{dx}{dt} = 2t\left(4-t^2\right)^{1/2} - t^3\left(4-t^2\right)^{-1/2}$ d. $3/23$

3. a. $y = 2x - 4$ c. Both distances are 5.

d. Yes. Both distances are $1 + \dfrac{a^2}{4}$. See if you can use this to explain the reflection property of a parabolic mirror. If a light source is at the focus, all reflected rays are parallel to the axis.

4. a. $1/8$ b. $1/12$