Name________________________________ 11 October 2001

Teaching Assistant_____________________ Hour Test 2

Math 1501C Andrew

Instructions: 1. Closed book, calculators may be used.
2. Show your work and explain your answers and reasoning.
3. Express your answers in simplified form.

1. (25) Compute

a. $\lim_{n\to\infty} \dfrac{n^2 + (-1)^n n}{2n^2}$ b. $\lim_{n\to\infty} \left(1 + \dfrac{-3}{n}\right)^{\frac{n}{2}}$

c. y', y'', and $y'' + 6y' + 9y$ for $y = xe^{-3x}$

2. (25) Let $p(x) = 3x^4 - 20x^3 + 36x^2 - 1$.

a. Use the intermediate value theorem to show that p has at least one root in the interval [0,2].

b. Show that $p'(x) > 0$ for all x in the interval (0,2).

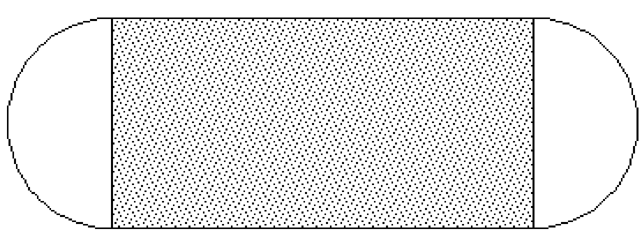
c. Deduce from parts a and b that p has *exactly* one root in the interval [0,2].

d. We will use Newton's method, starting with $x_1 = 1$ to approximate this root. Calculate x_2, expressing your answer *without* decimals.

3. (25) Consider the function $f(x) = \sin(x) + \dfrac{x}{2}$ defined on the interval $\left[0, \dfrac{3\pi}{2}\right]$.

a. Find the intervals on which f is increasing.
b. Find the intervals on which f is decreasing.
c. Find the intervals on which f is concave up.
d. Find the intervals on which f is concave down.
e. Find the largest and smallest values f assumes on the interval $\left[0, \dfrac{3\pi}{2}\right]$.

4. (25) A race track has two parallel straightaways of equal length, connected by semicircles, as shown below. If the length of the racetrack is 2.5 miles, what is the largest possible area of the rectangular infield (shaded below) that the track can enclose.



Answers

1. a $\frac{1}{2}$ b. $e^{-\frac{3}{2}}$

   c. $y' = e^{-3x}(1 - 3x)$, $y'' = -3e^{-3x}(2 - 3x)$, $y'' + 6y' + 9y = 0$.

2. a. *p* is continuous, *p*(0) < 0, *p*(2) > 0, so by the Intermediate Value Theorem, there exists *c* in (0,2) with *p*(*c*) = 0.

   c. First solution: Since $p'(x) > 0$ on $(0,2)$, $x_1 < x_2$ implies $f(x_1) < f(x_2)$. Consequently, $x_1 \neq x_2$ $f(x_1) \neq f(x_2)$, so there is only one c with $f(c) = 0$.

   Second solution: If there are $x_1 < x_2$ with $f(x_1) = f(x_2) = 0$, then by Rolle's Theorem, there is d, $x_1 < d < x_2$ with $f'(d) = 0$, contradicting the result of part b.

   d. $x_2 = \frac{1}{4}$

3. a. *f* is increasing on $\left[0, \frac{2\pi}{3}\right]$ and on $\left[\frac{4\pi}{3}, \frac{3\pi}{2}\right]$

   b. *f* is decreasing on $\left[\frac{2\pi}{3}, \frac{4\pi}{3}\right]$

   c. *f* is concave up on $\left[\pi, \frac{3\pi}{2}\right]$

   d. *f* is concave down on $[0, \pi]$

   e. Absolute maximum is $f\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2} + \frac{\pi}{3}$. Absolute minimum is *f*(0) = 0.

4. $\frac{25}{32\pi}$