MATH 2401, SOLUTIONS OF PRACTICE TEST 1

1

$$(\mathbf{u}(t)\cdot\mathbf{v}(t)) = \cos^2 t - \sin^2 t + t^2 = \cos 2t + t^2$$

Thus

$$\frac{d}{dt}(\mathbf{u}\cdot\mathbf{v}) = -2\sin 2t + 2t$$

and

$$\frac{d}{dt}f(t)\mathbf{u}(t) = e^{2t}\left((2\cos t - \sin t)\,\mathbf{i} + (2\sin t + \cos t)\,\mathbf{j} + (2t+1)\,\mathbf{k}\right)$$

2

$$\|\mathbf{r}'(t)\| = 6\sqrt{1+2t^2+t^4} = 6(1+t^2)$$

The length is given by

$$\int_0^1 6(1+t^2)dt = 6\left(1+\frac{1}{3}\right) = 8$$

3 The speed is given by

$$v(t) = \|\mathbf{r}'(t)\| = \sqrt{34+(-4t+3)^2} = \sqrt{16t^2-24t+43}$$

For the minimum we must solve $v'(t) = 0$ that gives $32t - 24 = 0$ ot $t = 3/4$. The value of the speed at $t = 3/4$ is 34.

4

$$\mathbf{T}(t) = \frac{-6\sin 2t\,\mathbf{i} + 6\cos 2t\,\mathbf{j} + \mathbf{k}}{\sqrt{36\sin^2 2t + 36\cos^2 2t + 1}} = \frac{1}{\sqrt{37}}(-6\sin 2t\,\mathbf{i} + 6\cos 2t\,\mathbf{j} + \mathbf{k})$$

$$s'(t) = \|\mathbf{r}'(t)\| = \sqrt{37}$$

Thus

$$\kappa(t) = \frac{\|\mathbf{T}'(t)\|}{\sqrt{37}} = \frac{\sqrt{12^2}}{37} = \frac{12}{37}$$

5 The domain of $f(x,y,z)$ is

$$\{(x,y,z)\,|\,x^2+y^2>1 \text{ and } z>0\}$$

while the range is all $R$ (all real numbers). The equation required is $\sqrt{x^2+y^2-1} - \ln z = \sqrt{1^2+2^2-1} - \ln e^2 = 0$ so that the equation is

$$\sqrt{x^2+y^2-1} = \ln z$$

6 $|xy| \leq \frac{1}{2}(x^2+y^2)$ so that

$$\left|\frac{xy}{\sqrt{x^2+y^2}}\right| \leq \frac{1}{2}\sqrt{x^2+y^2}$$

so that

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}} = 0$$

On the other side calling

$$f(x,y) = \frac{xy}{\sqrt{x^2+y^2}}$$

we get

$$\lim_{x\to 0} f(x,0) = 0$$

while

$$\lim_{x\to 0} f(x,x) = \lim_{x\to 0} \frac{x^3}{\sqrt{x^7}} = \infty$$

so that the limit does not exists

7

$$\begin{aligned}\frac{\partial^2}{\partial x \partial y}(y\cos y + xy\sin xy) =& \frac{\partial}{\partial y}\left(\frac{\partial}{\partial x}(y\cos y + xy\sin xy)\right) = \\ =& \frac{\partial}{\partial y}(xy^2\cos xy + y\sin xy) = \\ =& -x^2y^2\sin xy + 2xy\cos xy + \sin xy + xy\cos xy\end{aligned}$$