n 59

Let $Y = X_1 + X_2 + X_3$

$$Y \simeq N(180, 45)$$

$$P(Y \le 200) = \Phi\left(\frac{200-180}{\sqrt{45}}\right) =$$

$$\Phi\left(\frac{20}{6.71}\right) = \Phi(2.98) = 0.9986$$

b) $\bar{X} \simeq N(60, 5)$

$$P(\bar{X} \geq 55) = 1 - P(\bar{X} \le 55) =$$

$$= 1 - \Phi\left(\frac{55-60}{\sqrt{5}}\right) = 1 - \Phi(-2.24) =$$

$$= 0.9875$$

c) Let $Z = X_1 - 0.5X_2 - 0.5X_3$

$$E(Z) = E(X_1) - 0.5E(X_2) - 0.5E(X_3) = 0$$

$$V(Z) = V(X_1) + 0.5^2V(X_2) + 0.5^2V(X_3) =$$

$$\frac{3}{2}15 = 22.5$$

$$P(-10 \le Z \le 5) = \Phi\left(\frac{5}{\sqrt{22.5}}\right) - \Phi\left(\frac{-10}{\sqrt{22.5}}\right)$$

d

$$X_1 + X_2 + X_3 \simeq N(150, 36)$$

$$P(X_1 + X_2 + X_3 \leq 160) = \phi\left(\frac{160-150}{6}\right)$$

$$P(X_1 + X_2 \geqslant 2X_3) = P(X_1 + X_2 - 2X_3 \geqslant 0) =$$

$$X_1 + X_2 - 2X_3 \simeq N(-30, 50)$$

$$= \phi\left(\frac{0+30}{\sqrt{50}}\right) = \phi\left(\frac{30}{\sqrt{50}}\right)$$

n64

a) $\cancel{X_1}\ \cancel{X_3}\ \cancel{X_5}\ \cancel{X_7}\ \cancel{X_9}$

$E(X_1 + X_2 + \dots X_{10}) = 5E(X_1) + 5E(X_2) =$

X_i = morning waiting time if i odd

X_i = evening waiting time if i even

$= 5 \cdot 4 + 5 \cdot 5 = 45$

b)

$V(X_1 + X_2 + \dots X_{10}) = 5V(X_1) + 5V(X_2) =$

$5\,\frac{8}{12} + 5\,\frac{10}{12} = \frac{20}{12} = \frac{15}{2}$

c)

$E(X_1 - X_2) = E(X_1) - E(X_2) = 4 - 5 = -1$

$V(X_1 - X_2) = V(X_1) + V(X_2) = \frac{3}{2}$

d) ~~B~~ it just 5 time the previous answer

~~Ell~~ average $= -5$

Variance $= \frac{15}{2}$

n 69

a) $E(X_1+X_2+X_3) = E(X_1)+E(X_2)+E(X_3) =$

$= 800+1000+600 = 2400$

b) $V(X_1+X_2+X_3) = V(X_1)+V(X_2)+V(X_3) =$

if X_1, X_2, X_3 are independent.

$= 16^2+25^2+18^2 =$

c) $V(X_1+X_2+X_3) = V(X_1)+V(X_2)+V(X_3)+$

$\operatorname{coV}(X_1,X_2)+\operatorname{cov}(X_1,X_3)+$

$\operatorname{cov}(X_2,X_3) =$

$16^2+25^2+18^2+80+90+100.$

n 37

$\bar{X} = \frac{1}{2}(X_1 + X_2)$

$p(x)$ p.d.f of $\bar{X}$

$p(25) = P(X_1 = 25 \cap X_2 = 25) = 0.2 \cdot 0.2 = 0.04$

$p(32.5) = P(25, 40) + P(40, 25) = 2 \cdot 0.2 \cdot 0.5 = 0.2$

$p(40) = P(40, 40) = 0.5 \cdot 0.5 = 0.25$

$p(45) = P(25, 65) + P(65, 25) = 2 \cdot 0.2 \cdot 0.3 = 0.12$

$p(52.5) = P(40, 65) + P(65, 40) = 2 \cdot 0.3 \cdot 0.5 = 0.3$

$p(65) = P(65, 65) = 0.3 \cdot 0.3 = 0.09$

$E(\bar{X}) = 0.04 \cdot 25 + 32.5 \cdot 0.2 + 40 \cdot 0.25 +$

$45 \cdot 0.12 + 50.5 \cdot 0.3 + 65 \cdot 0.09 = 44.5$

$E(\bar{X}) = \mu$

n 40

a) $P(X=0) = P(X_1=0 \cap \cancel{\ } X_2=0 \cap X_3=0) = (0.5)^3$

$P(X=5) = P(X_1<10 \cap X_2<10 \cap X_3<10) -$

$P(X_1=0 \cap X_2=0 \cap X_3=0) =$

$(0.8)^3 - (0.5)^3$

$P(X=10) = 1 - P(X=5) \cdot P(X=0) = 1-(0.8)^3$

b) Generate a random number in ~~[0, 1, 10]~~ [0, 5, 10] n Times and compute The maximum among The n. Repeat as many Time a possible.

Clearly The larger n The larger The probability To get at least a 10.

Given n we have

$P(X=10) = 1 - P(\text{all } X_i < 10) = 1-(0.8)^n \xrightarrow[n \text{ large}]{} 1$

n 41

a)
eT p.m.f of X $\quad m(X)$

$$m(1) = p(1)\,p(1) = (0.4)^2$$

$$m(1.5) = 2\,p(2)\,p(1) = 2\cdot 0.4\cdot 0.3$$

$$m(2) = 2\,p(3)\,p(1) + p(2)\,p(2) = 2\cdot 0.2\cdot 0.4 + 0.3\cdot 0.3$$

$$m(2.5) = 2\,p(4)\,p(1) + 2\,p(3)\,p(2) = 2\,(0.1\cdot 0.4 + 0.3\cdot 0.2)$$

$$m(3) = 2\,p(4)\,p(2) + p(3)\,p(3) = 2\cdot 0.1\cdot 0.3 + 0.2^2$$

$$m(3.5) = 2\,p(4)\,p(3) = 2\cdot 0.1\cdot 0.2$$

$$m(4) = p(4)\,p(4) = 0.1^2$$

b) $$P(X < 2.5) = (0.4)^2 + 2\cdot 0.4\cdot 0.3 + 2\cdot 0.2\cdot 0.4 + 0.3^2$$

c) $X_1 - X_2$ $\quad n(x)$ $\quad$ p.m.f.

$$n(0) = p(1)^2 + p(2)^2 + p(3)^2 + p(4)^2$$

$$n(1) = 2\,p(1)\,p(2) + 2\,p(2)\,p(3) + 2\,p(3)\,p(4)$$

$$n(2) = 2\,p(1)\,p(3) + 2\,p(2)\,p(4)$$

$$n(3) = 2\,p(1)\,p(4)$$

d)

$$P(\bar{X} \leq 1.5) = p(1)^4 + 4\,p(2)\,p(1)^3 + 4\,p(3)\,p(1)^3 + 6\,p(2)^2\,p(1)^2$$

n 46

a) $\bar{X}$ is centerd on 12 cm and has s. d. $\frac{0.4}{4} = 0.1$

b) $\bar{X}$ is centered on 12 cm and has s.d. $\frac{0.4}{8} = 0.05$

c) The second b/c The variance is smaller

n 48

a) $\bar{X} \simeq N(50, 0.01)$

$P(49.75 \leq \bar{X} \leq 50.25) =$

$\phi(2.5) - \phi(-2.5) =$

$0.9938 - 0.0062 =$

b) $\phi\left(\frac{50.25 - 49.8}{0.1}\right) - \phi\left(\frac{49.75 - 49.8}{0.1}\right) =$

$\phi(4.5) - \phi(-0.5) = 1 - 0.3085$

n 51

$\bar{X}$ sample average 1st day

$\bar{Y}$ sample average 2nd day

$P(\bar{X} < 11 \cap \bar{Y} < 11) = P(\bar{X} < 11)\, P(\bar{Y} < 11)$

$\bar{X} \simeq N(10, 4/5)$

$\bar{Y} \simeq N(10, 4/6)$

$\Phi(1.09)$

$P(\bar{X} < 11) = \Phi\left(\frac{11-10}{\sqrt{0.8}}\right) = = 0.8621$

$P(\bar{Y} < 11) = \Phi\left(\frac{11-10}{\sqrt{4/6}}\right) = \Phi(1.22) : 0.8888$

$\text{Prob} = = 0.8621 \cdot 0.8888 = 0.7662$

n 53

$\bar{X} \simeq N(50, 1.44/9)$

$P(\bar{X} \geq 51) = 1 - P(\bar{X} \leq 51) = 1 - \Phi\left(\frac{51-50}{1.2/3}\right) =$

$1 - \Phi\left(\frac{3}{1.2}\right) = 1 - \Phi(2.5) = 0.0062$

b) $\bar{X} \approx N\left(50, \frac{1.44}{40}\right)$

$$P(\bar{X} \geq 51) = 1 - P(\bar{X} \leq 51) = 1 - \Phi\left(\frac{(51-50)2\sqrt{10}}{1.2}\right) =$$

$$1 - \Phi(5.2) = 0$$

n 55

T_0 = # of student with no errors

$T_0 \approx N(20, 12)$

a) $P(T_0 \geq 25) = 1 - P(T_0 \leq 24) = \cancel{1 - \Phi\left(\frac{24+0.5-20}{\sqrt{12}}\right)}$

$$1 - \Phi\left(\frac{24 + 0.5 - 20}{\sqrt{12}}\right) = 1 - \Phi(1.3)$$

b) $P(15 \leq T_0 \leq 25) = \Phi\left(\frac{25+0.5-20}{\sqrt{12}}\right) -$

$$-\Phi\left(\frac{14+0.5-20}{\sqrt{12}}\right)$$

n 58

a) ~~Ell~~ Let $V = 27X_1 + 125X_2 + 512X_3$

Then

$$E(V) = 27E(X_1) + 125E(X_2) + 512E(X_3) =$$

$$27 \cdot 200 + 125 \cdot 250 + 512 \cdot 100$$

$$V(V) = 27^2 V(X_1) + 125^2 V(X_2) + 512^2 V(X_3)$$

$$= 27^2 \cdot 10^2 + 125^2 \cdot 12^2 + 512 \cdot 8^2$$

b) If they are not independent

$E(V)$ is still correct

$V(V)$ is correct only if

$cov(X_i, X_j) = 0 \quad i \neq j$